Engineering Design-Based Mathematics Activities: Ratio and Proportion Sample

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ABSTRACT

In STEM education, the engineering design process serves as a pedagogical strategy to connect science and mathematics concepts. While STEM approaches are increasingly favored in science courses, the role of mathematics in integrated STEM activities is often overlooked, despite its foundational importance across STEM disciplines. This manuscript investigates the implementation of engineering design activities in a middle school mathematics course using an integrated STEM approach. Conducted during the 2022-2023 academic year, the study involved 21 seventh-grade students from a private school in Turkey's Black Sea region. The activities were designed for the learning outcomes of the seventh grade Ratio and Proportion subject. The activities required students to apply ratio and proportion concepts as integral components of the engineering design process. At the end of the process, it was observed that the activities could support the concepts of ratio and proportion from the perspectives targeted in the learning outcomes, students were able to effectively use the content related to mathematics and science for design, and their active participation in the lesson increased.

Keywords: Engineering design process, mathematics activity, integrated STEM education, middle school

Mühendislik Tasarım Temelli Matematik Etkinlikleri: Oran ve Orantı Örneği

ÖZ

STEM eğitiminde mühendislik tasarım süreci, fen ve matematik kavramlarını birbirine bağlamak için pedagojik bir strateji olarak görülmektedir. STEM yaklaşımı Fen derslerinde daha fazla tercih edilirken, STEM disiplinleri arasındaki temel önemine rağmen, matematiğin bütünleşik STEM etkinliklerindeki rolü genellikle göz ardı edilmektedir. Bu araştırma, bütünleşik STEM yaklaşımına göre yürütülen bir ortaokul matematik dersinde uygulanan mühendislik tasarım etkinliklerini ve uygulama süreçlerini incelemeyi amaçlamaktadır. 2022-2023 eğitim-öğretim yılında gerçekleştirilen çalışmaya, Türkiye'nin Karadeniz bölgesindeki bir özel okuldan 21 yedinci sınıf öğrencisi katılmıştır. Etkinlikler, yedinci sınıf Oran ve Orantı konusunun kazanımlarına yönelik olarak tasarlanmıştır. Etkinlikler, öğrencilerin oran ve orantı kavramlarını mühendislik tasarım sürecinin ayrılmaz bileşenleri olarak uygulamalarını gerektirmiştir. Süreç sonunda, etkinliklerin oran ve orantı kavramlarını kazanımlarda hedeflenen açılardan destekleyebildiği, öğrencilerin matematik ve fen bilimleri ile ilgili içeriği tasarım için etkin bir şekilde kullanabildikleri ve derse aktif katılımlarının arttığı gözlemlenmiştir.

Anahtar kelimeler: mühendislik tasarım döngüsü, matematik etkinliği, bütünleşik STEM eğitimi, ortaokul

Article Information:

Submitted: 26.05.2023 Revised: 01.03.2025 Accepted: 04.03.2025

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INTRODUCTION

Science, technology, engineering, and mathematics (STEM) education includes hands-on activities that prepare students for the challenges they may face in the complex world and developments of the new competitive era (Hong, 2017; Wahono et al., 2020). One of the most prominent features of STEM education is the integration of two or more disciplines (Shaughnessy, 2013; Tsupros et al., 2009). This approach, which makes meaningful connections between different STEM disciplines and explores the relationships and educational processes between them, is referred to as integrated STEM education (Sanders, 2009). The integration process in integrated STEM education is realized through the presentation of content and concepts in the context of a problem, project, or task (Nadelson & Seifert, 2017). In this context, the ultimate goal of integrated STEM education is to concretize abstract concepts, theories, and formulas by reconstructing interdisciplinary knowledge in a real-world setting. Knowledge from everyday life can serve as a bridge between learners and the outside world as long as it is applied in everyday situations (Di et al., 2021).

The goals of designing through applied technology and engineering activities to use interdisciplinary knowledge have become common in K-12 education STEM (Arık & Topçu, 2020). In these educational , engineering design is considered as a pedagogical strategy to make connections to science or mathematics concepts and practices (Mohd Shahali et al., 2017; Wang et al., 2011). Incorporating engineering disciplines into the educational process through design practices contributes to the development of skills such as problem solving, creative thinking, communication and collaboration, and mathematical knowledge (Brophy et al., 2008; National Research Council, 2009).

Design is the fundamental problem-solving approach of the engineering discipline. The Engineering Design Process (EDP) refers to all the steps that guide engineers in solving a problem (Dym, 1999). According to National Academy of Engineering and National Research Council (2009), the inclusion of engineering at the K-12 level should support and emphasize the EDP. The Engineering Design Process is that triggers curiosity and creativity, aims for an optimal solution rather than a single correct solution, and combines technical knowledge and personal preferences to this end. It refers to a systematic progression that proceeds cyclically according to specific goals and criteria (Dym, 1999; Hynes et al., 2011). Various models for the Engineering Design Process at different levels have been proposed in the literature. The cycle shown in Figure 1 is the process defined by the Next Generation Science Standarts (Next Generation Science Standarts [NGSS] Lead States, 2013) for the middle school level. This cycle includes 1) looking at the problem holistically, 2) evaluating different solutions and combining them when necessary, and 3) deciding on the most appropriate solution at the end of the iterative process. Due to the cyclical nature of the process, a clear, sequential progression is not mandatory. Students following this process can go back at any step to generate solutions to the problem and repeat the process.

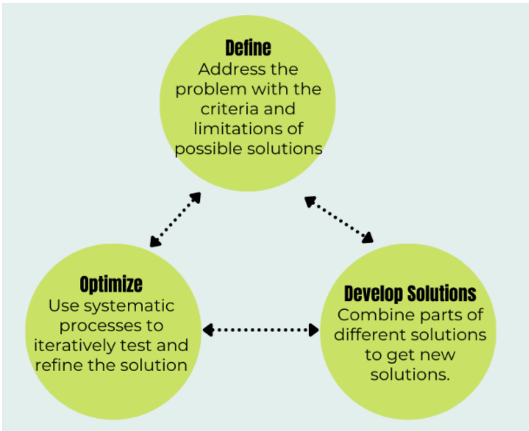


Figure 1. Engineering Design Process (NGSS, 2013)

In the literature, STEM education and engineering integration have been in high demand since their emergence, but how STEM teachers design and implement learning activities is still a big question that needs to be addressed (Gyasi et al., 2021). Furthermore, despite studies conducted by educational researchers to explain integrated STEM education, there is a need for teacher guides (Dare et al., 2019). For this reason, this study aims to investigate the construction activities and implementation processes developed for the outcomes of the Ratio and Proportion topic in seventh grade mathematics classrooms. Construction activities provide opportunities to present mathematical concepts and questions as natural elements of the problem-solving process. In this study, the concepts of ratio and proportion were used in meaningful contexts and applications for both mathematics and engineering.

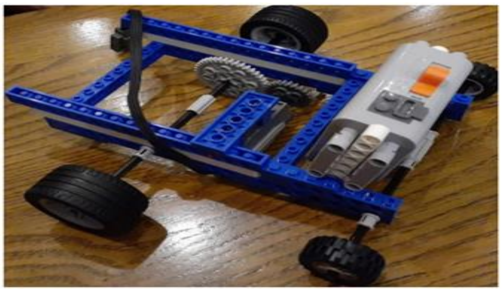
IMPLEMENTATION OF THE ACTIVITIES

Twenty-one seventh-grade students from a private middle school participated in the implementation of this study during the 2022-2023 school year. The activities applied in the study were designed by the researchers for the seventh grade Ratio and Proportion subject acquisitions. STEM activities should involve problems to explore and understand the real world, emphasize various STEM disciplines. They should also include STEM integration and engineering design and support students' active participation (Okulu & Oguz-Unver, 2021). These criteria were taken into account when designing the activities. In addition, it is also aimed to be suitable for the achievements related to the subject in the curriculum (Ministry of National Education, 2018). The activities were designed so that students would use the mathematical concepts of ratio and proportion as natural elements of the design process when applying the steps of the Engineering Design Process. Each activity was conducted in three 40-minute class periods, one week apart.

Activity 1: Two Gear Activity

In this activity, students were expected to recognize the effects of gears used in vehicles such as bicycles, cars, cranes, and elevators on speed and moment and to create the most appropriate designs for gears in a vehicle. It is aimed to use the concepts of ratio and proportion and to follow the EDP in order to reach the design. This activity is related to engineering in that gears are often used to convert the power generated in an engine into the desired ratio of moment and speed.

The materials needed for this activity are the LEGO 9686- Simple and Motorized Machines Set, a RPM meter (tachometer), and Worksheet Two Gear Activity (Appendix 1). The LEGO set was used to build model vehicles and gears. In this activity, a model vehicle was created for each group (see Photograph 1) and each group was given a gear from the LEGO set with 8, 24, and 40 teeth.



Photograph 1. Model vehicle created for the activity

Activity 1 Introduction Phase

In this phase, students were asked if they used bicycles and what characteristics they expected when selecting their bicycles so that they would be aware of the problem for which they would seek solutions. Most of the students indicated that they use bicycles. They indicated that they would look for features such as "good looks," "size," "comfort," and "gears" when selecting their bicycles. After these answers, the researcher focused on the answers about comfortable driving and gears. He asked how comfortable driving can be achieved, what gears do in vehicles, and how vehicles accelerate and decelerate. It was found that the students knew the relationship between comfortable driving and speed with gears, but they had no idea how the transmission works. After discussion, the principles of operation and use of the gear systems that make the transmission work, the place of the transmission in engineering, and the principles of power and speed transmission of these systems were explained. At this stage, it was explained that gears are generally assembled into gear mechanisms and that these mechanisms are used in some vehicles that are commonly encountered in daily life. In these mechanisms, speed and moment are changed in different ways by connecting gears of different sizes. The way a gear system transmits motion is determined by what is called the gear ratio. This ratio is calculated as follows (Cürgül, 2020):

$$\text{Gear ratio} = \frac{\textit{Number of driven gear teeth}}{\textit{Number of driving gear teeth}} = \frac{\textit{Input speed}}{\textit{Output speed}}$$

The motion transfer of the system is related to whether this ratio is less than or greater than 1, but this was not explained to the students at this stage. It is explained that only the gear ratio can be calculated with this formula. It was expected that this would be discussed and interpreted in the application phase.

To explain the relationship between speed and moment in gear systems, students were given examples of bicycles and kitchen mixers. For example, if you are riding a bicycle uphill, when you shift down the gear, the speed decreases and the moment increases. This makes it easier to climb the hill. Similarly, when whipping a thick mass with a mixer, the mass is whipped more easily at a low setting.

Activity 1 Implementation Phase

Students were divided into groups of four (Due to the number of participants, there were 5 students in one group.) according to class size and number of model vehicles. Work materials were distributed to each group and the following three tasks were given:

"Problem 1: You are to design the accelerator gear for the transmission of a vehicle. Using the gears available to you, find the pair of gears that will accelerate this vehicle to the highest possible speed and determine the ratio between the gears.

Problem 2: You are asked to design a deceleration gear for the transmission of a vehicle. Using the gears available to you, find the pair of gears that will accelerate this vehicle to the lowest possible speed and determine the ratio between them.

Problem 3: Since the vehicles used in everyday life are heavy, they need to have high moment so they do not stall when starting up or climbing a steep road. Using the gears available to you, find the pair of gears that will give this vehicle the highest possible moment and determine the ratio between the two gears."

In order to solve design problems, the scope of the problem must first be determined and information gathered accordingly (NGSS, 2013). For this reason, students were asked the following questions, "*What is required of you in the problems, are there any particular problems or constraints on the design?*"

Student 1: We were asked to design gears that accelerate and decelerate a vehicle.
Student 2: We were also asked to design a transmission for starting and climbing. Like the first gear of a car.
Student 3: As far as I understand, we are supposed to make these designs with the materials provided to us.
Researcher: Yes. Do you have any idea what you can come up with for a solution?
Student 1: We can try the gears we have one by one. We can find the fastest one, the slowest one, or all of them.
Researcher: Do you mean we can put the gears in the car and try them out to see the effects?
Student 1: Yes, we can see it that way.
Student 1: We can also use the translation ratio we just saw.
Researcher: To find the best solutions to the problems, you can both calculate the gear ratios and compare the results by experimenting on the vehicle. Remember to record each experiment and procedure on the worksheet.

After the discussion, the groups began experimenting with the solution. The groups chose two gears, one rotating and one driven. They calculated the gear ratio for the chosen combination and the corresponding output speed and wrote them down on the worksheet. To compare the data, they connected the gears to the vehicle and had the opportunity to both measure the output speed with a tachometer and observe the rotational speed. Since the same engine was connected to all of the vehicles used as models in this design, the input speed was written the same and given to the students in advance. At this stage, the researcher helped each group connect the gears to the motor and the output arm and measure the speed with a tachometer. The same procedure was repeated for all combinations. Meanwhile, it was interpreted how the speed was affected depending on whether the gear ratio was greater or less than 1. The following is an excerpt from the discussion of this.
Researcher: You did some calculations on the gear ratio and saw the effects. What were the effects of choosing the gears as rotating and connecting them?

To answer this question, the groups analyzed the data they had written down on the worksheet.

Student 6: If the ratio increases, the speed decreases.
Student: When the gear ratio is large, the speed is low; when it is small, the speed is high.
Researcher: Yes, in gear mechanisms,

depending on whether the gear ratio is larger or smaller than 1, the system can decrease or increase the speed. When the ratio is less than 1, the motion is transmitted by increasing the speed, when it is greater than 1, it is transmitted by decreasing the speed, and when it is equal to 1, it is transmitted exactly the same.
Student 7: We could have used this information directly if we had known it at the beginning of the draft.
After the discussion, the observations and calculations to solve the last problem were also done with the weight attached to the back of the vehicle. In this phase, the groups worked both within their own group and in collaboration with other groups. Once the groups decided on the designs they thought would work best for solving all three problems, they presented them in the classroom and the final phase began.

Activity 1 Result Phase

In this phase, some questions were asked to discuss with the students how they decide the equality of two ratios, how they find the value of the other ratio given one of the two multiplications, and how they explain situations with direct and inverse ratios. Below is an excerpt from these discussions:
Researcher: You calculated separately the ratio between the number of teeth of the gears, as well as the ratio between the input speed and the output speed of the motor. How did you determine that these ratios were equal?
Student 8: We made a simplification, we said that they are proportional when the same rational numbers come.
Student 7: It can also be an extension. If the same thing comes out in both cases, the ratios are the same.
Researcher: After you changed the gears and measured the input speed, how did you find the output speed? What do you do to find the other one when you have one of the two values whose ratio you know?
Student 9: If we know the ratio, it shows us how two multiplicities are distributed. For example, in this task, we knew that the ratio was 1/5. Since we knew the input velocity was 550, we multiplied inside and outside. We could have also multiplied the numerator and denominator separately and found the ratio by expanding.
Researcher: Mathematically, what does it mean that the ratio of the number of teeth of the gears and the ratio of the engine speeds are the same? How do you conclude that the two ratios form a ratio?
Students: We can say that they are proportional.
Researcher: Did you notice the relationship between the moment and the speed of your vehicle? How would you explain this relationship?
Student 3: As the speed of the car decreased, it became easier to pull the load. Just like a bicycle.
Student: The moment and speed of the car are inversely proportional.
After the discussion, the researcher summarized what was done in the activity and the mathematical ideas that were reached. The basic ideas are the following: A gear ratio calculated as 1/5 does not only express a simple fraction. Mathematically, it also provides a proportional situation. For example, if the torque increases by a factor of 5, the speed decreases in the same proportion. The effects of changes to the gear system on the gear ratio can be explained by the concepts of direct and inverse proportionality. The solution development and optimization phases of the EDP required the calculation of gear ratios. Students compared the results of different ratios to improve their designs. The concepts of ratio and proportion explain the working principle of vehicles such as cars and bicycles in daily life. Afterward, the activity was completed.

Activity 2: Hydraulic Leverage Activity

In this activity, students designed a fluid-powered lever, similar to systems in which fluid is used as a motive force. It is aimed for students to use the concepts of ratio and proportion and EDP while creating a model of the designs to lift the maximum load with the least force. This activity is related to engineering in that Pascal's Law is often used in the manufacture and operation of fluid-powered systems such as hydraulic jacks, braking systems, and construction equipment (Halliday et al., 2014).

The materials needed for the activity are syringes of various sizes, clear tubing, weight disks, a chipboard platform for each group, a colored fluid, and the Hydraulic Leverage Activity worksheet (Appendix 2). The cross-sectional areas of the syringes used for this activity and the weights of the disks were calculated in advance and communicated to the students. The particle board platform prepared for the activity was used so that the syringes could be easily attached and connected together. The setup of the activity is shown in Photograph 2.

Photograph 2. Hydraulic Leverage activity setup

Activity 2 Introduction Phase

Before we began the activity, students were asked how they set the height of objects such as a work chair/gaming chair and if they had ever seen how to change a car tire. In this section, the discussions revolved around the case of lifting a load with little force. Below is an excerpt from the discussions.

Student 10: I have a work chair. I adjust the height by pulling the lever under it.

Student 11: I have seen a jack used to change a car tire. There is a lever on the jacks like on the chair. When you push the lever, you can lift the car.

Researcher: Well, have you ever thought about how a small jack can do this, while ordinary people can not lift a car by themselves?

Student 5: Is the force transmitted?

Researcher: What is the transmitted force and how can it be transmitted?

The students thought about this question but could not give a clear answer. Then it was explained to the students that the change in pressure applied to a fluid in a closed container is transmitted in all directions of the fluid and this situation is called Pascal's Principle (Serway & Beichner, 2021). Then, examples of Pascal's principle were given from everyday life, such as perfume bottles, fire ladders, buckets, and jacks. It was mentioned that this principle is also used in engineering to design and use tools that allow heavy vehicles to be lifted with less force. Then it was explained to the students that, just like engineers, they would design a hydraulic lever using the change in fluid pressure and the task was given.

"Problem: You are to design a lever to lift vehicles arriving at an auto repair store with as little effort as possible. Using the materials at your disposal, design a model that will make this possible."

Activity 2 Implementation Phase

The students were first divided into groups of 4. Due to the number of participants, there were 5 students in one group. Before starting the design phase, the students were asked to understand the working principle of the hydraulic lever system and make their decisions accordingly. For this reason, the groups were asked to select two syringes each, attach them to the platform, fill the syringes with fluid, and connect them with a hose. Meanwhile, the cross-sectional areas of the selected syringes were recorded in the appropriate section of the worksheet. Students were asked to place a weight on one of the syringes to move the assembly. After the assembly moved, disks were added to the other syringe until the assembly reached equilibrium. The weights that brought the system to equilibrium were entered in the appropriate section of the worksheet.

According to Pascal's principle, the pressure created when a force (F) acts on a fluid over a given cross-sectional area (A) is equal to the force/cross-sectional area. To bring the system into equilibrium, this ratio must be the same on both sides of the apparatus; hence $\frac{F1}{A1} = \frac{F2}{A2}$ (Halliday et al., 2014).

The lever mechanism works on this principle. To help students see this connection, the following discussions were based on their initial experiments and data.

Researcher: What is the relationship between the cross-sectional areas of the syringes you chose for the platforms and the weights you placed on them?

Student: We need to put a larger weight on the syringe with a larger cross-sectional area.

Researcher: Examine the cross-sectional area of the first syringe and the weight you placed on it, and the cross-sectional area of the second syringe and the weight you placed on it. Looking at the weights and cross-sectional areas you used to bring the lever system into equilibrium, can you explain the condition necessary for the

system to remain in equilibrium?
Student: Ratios must be equal.
Researcher: Since the ratios on both sides of the system are equal, how would you express the relationship between these ratios?

After the discussions were completed and the functional logic of the lever system was understood, various experiments were conducted. These involved determining syringes and the weight to be placed on one side, first calculated on paper and then tested by inserting them into the apparatus. One of the applications that the students made in this process can be seen in Photograph 3.

Photograph 3. Hydraulic Leverage activity applications

After the experiments and researches carried out by the groups in order to comprehend the working principle of the apparatus, the problem was solved. For this purpose, the groups were asked to keep a weight constant and observe other variables. To solve the problem, the force should be transferred from the syringe with a small cross-sectional area to the syringe with a large cross-sectional area. At this stage, the groups were expected to reach this solution by conducting new experiments with the data they had previously recorded in the table. After all groups created their designs, the final phase was started.

Activity 2 Result Phase

In the concluding section, the following questions were asked and answered in order to discuss with the students how they explained the direct and inverse proportional situations through the activity:

Researcher: Can you explain the relationship between the force required to lift the weight and the cross-sectional area of the syringe containing the solid weight?
Student 6: We have chosen 18 N as the fixed weight. In the experiments, we saw that the thicker the syringe containing the fixed weight, the greater the force required to lift it. So these two factors were inversely proportional.
Researcher: Can you explain the relationship between this force and the cross-sectional area of the syringe to which it is applied?
Students: They are directly proportional.
Researcher: How did you arrive at the idea that the force required to lift the weight and the cross-sectional area of the syringe to which it is applied are directly proportional?
Student 5: As the cross-sectional area increases, the force also increases. That's why we said it was directly proportional. We can see that in the table.
Researcher: If you decrease the cross-sectional area of the syringe on which the force is applied, did you notice that the force you have to apply decreases by the same amount?
Student 12: Yes, each part of the system was related to each other. Either directly or inversely proportional.
Researcher: How much force would you have to apply if you wanted to lift an object 50 N with the syringes you used in your design?
The groups answered this question using the combination they chose for their design.

After discussion, the researcher summarized the following key ideas: In order to come up with a solution for this design, you first had to understand the working principle of the lever. You performed tests for this. For the design that produces the highest force, you calculated the ratio between the cross-sectional areas. Using this ratio, you were able to calculate how many times the force applied to the lever increased or decreased on the other side. You used the equation $\frac{F1}{A1} = \frac{F2}{A2}$ to generate different solutions and improve the design. This equation also determined how different cross-sectional areas affect the force distribution. As a result, you observed the advantages and disadvantages of increasing the force/sectional area ratio in a hydraulic system and created your designs accordingly. You were also able to see that each element of the system is directly or inversely proportional to each other. After this summary and evaluation, the activity was completed.

Activity 3- Designing a Life Center

In this activity, students designed a life center by considering various criteria such as time, cost, usability, and visual appropriateness. While transferring the designs to paper and calculating the time and cost required for the construction of the design, it was aimed to utilize the concepts of scale, ratio and proportion and to follow the EDP. This activity has to do with engineering because it involves transferring designs to paper by reducing or enlarging them to a specific scale and creating them with constraints such as cost, time, and ease of use.

The materials needed for the activity were the Life Center Activity Design Sheet (Appendix 3.), the Life Center Activity Worksheet (Appendix 4.), the Life Center Cost Sheet (Appendix 5.), a ruler, and colored pencils.

Activity 3 Introduction Phase

Before starting the activity, students were asked to explain why countries on a world map appear to cover different areas when examined. The students explained that the reason was that the areas and sizes of the countries were different. Then they were asked how they would explain the fact that our country occupies a small area on the world map, while on the map of Turkey it is shown so large that it covers the whole paper. Some students answered this question by saying that a world map and a map of Turkey were drawn by reducing them to different scales, while others mentioned that they were drawn with different scales.

After the discussion was completed, the concept of scale was explained to the students as "the relationship between the length given on a map, plan, model, or picture and the actual length." The benefits of using scale in engineering and in any drawing were cited as the ease of representing things that are too large or too small to represent on paper or with a model. Some scales were discussed in terms of the length of the map and the actual length. For example, it was recalled that 1 cm length on a 1/100 000 scale map represents 100 000 cm, while 1 cm length on a 1/500 scale map actually corresponds to 500 cm. Then the problem was presented to explain the design task:

"Problem: We are looking for a team to build a life center on a vacant lot to be developed for a new housing development. The team with the best design according to the given conditions will win and the life center will be built according to that team's design. Determine the structures you would like to have in your life center and transfer your design to the plan. Remember, when making your choice, you must consider the time and cost required to implement your design."

Activity 3 Implementation Phase

The implementation phase began with discussions for the students to determine the scope of the problem. We reproduce an excerpt from these discussions:
Researcher: what is required for the problem? What are the things you need to do and consider for the design?
Student 7: We are asked to design a life center. The construction time of the design should not be too long and we need to pay attention to the price.
Researcher: In your opinion, are there any other features that the design of a life center should have in order to be considered successful?
Student 13: The streets and settlements should be well designed. For example, it should be easily accessible everywhere.
Student 9: Since it is a life center, there should be buildings to meet all needs.

Considering these discussions, students were asked to think about the buildings that should be present in a life center to reveal their ideas in line with the purpose of the design task. All groups brainstormed among themselves to determine the structures they thought should be in the life center and wrote all ideas in the appropriate section of the worksheet. Then the design phase began.

A specific scale was used in the draft sheet created in advance. Students must find this scale to calculate the time and cost of the design they will create.
Researcher: You have determined the structures to be included in your design. Do you have enough information to calculate the time and cost to build these structures?
Student: We can use the time and cost table.
Researcher: The time-cost table has data for a m^2 *dimension. Would it be appropriate to use*

this data for your design?
Student 10: Then we will adjust the dimensions of the design to match them.
Student 2: We can calculate how big the drawings really are by using a scale.

For the scale calculation, the actual dimensions of the forested area were given on the design paper and it was explained that this area would not be touched. Students measured the squares on the draft paper to the nearest centimeter with their rulers and calculated the scale. (It was found that the actual length of the forested area was 400 m x 500 m and 200000 m^2. Since it was drawn on the plan as 4cmx5cm=20 cm^2, the scale is 1/10000).

The groups discussed the structures they had previously thought of, prioritized them, and created their designs, taking into account the limited space, cost, and total time needed. They also used the schedule to determine the time and cost required to build the final version of their designs. Students were also advised that when calculating construction time, it should be assumed that construction of one structure would begin after the completion of the other. An example of the groups' design process can be found in Photograph 4.

Photograph 4. Life Center design process

Once the designs were completed, all groups shared their designs, costs, and time needed for construction in the classroom. In the meantime, the groups discussed the pros and cons of all the designs. After the presentations, the best design was selected together with the class. Here is an excerpt from the discussions that took place during the presentations of the designs:
Student 12: Our design was a little more expensive than the other group's. But we think that our design is a complete life center. There are enough houses and we have set up places where the residents can satisfy all their needs, even if they never leave the house. We have a school, cafes, a hospital, a gym, a stationery store, a hairdresser, and even a small shopping center. We also have wide streets. It's easy to move around the life center.
Student 4: In your design, you put a paved road around the forest. I do not think that's a good thing. You see, we put a nursing home next to the forest in our design. We made good use of that area.
Student 6: It also takes a long time to build that.
Student 1: It looks like it was built in small areas to reduce the cost of the buildings, but you already put a shopping center there. A lot of the buildings in the design could have fit in it. Instead, more necessary structures could have been spread out over large areas.

Activity 3 Result Phase

In this phase, students were first asked what they take as a criterion when creating a design and what they primarily look for. Most students answered that they look for structures that can meet daily needs. One student said that he, together with his group mates, pays attention to create a design that fully meets all the required characteristics.

The question "*How did you calculate the cost of your design?*" was asked to discuss how students used scales and unit prices to figure out how much unit-priced structures cost with dimensions in the design. Most students indicated that they made a direct relationship between price and quantity.

To discuss how students expressed the relationship between two proportional multiplications and how they decided that two multiplications are proportional: "*What was the relationship between the area of the building unit you used or the number of stories to build and its cost? How did you decide on that ratio?*" was asked. The students realized that as the size and number of stories of the buildings increased, their cost increased at the same rate.

Discuss how students expressed the relationships between direct and inversely proportional multiplications and how they decided that two multiplications are inversely proportional: "*How did the number of workers you planned in your design affect the cost and time?*". The students said that increasing the number of workers increased cost and decreased time; therefore, it was directly proportional to cost and inversely proportional to time.

The activity concluded with a discussion of what they would change if they wanted to go back and change their designs. The groups answered this question by considering the criticisms they had received during the presentation of their designs and the flaws they had found in their designs compared to other presentations. For example, "we would evaluate the design space better," "we would place the structures more regularly," "we would not consider only cost/time/benefit/visuality," "we would plan better before we start drawing".

CONCLUSION AND RECOMMENDATIONS

In this manuscript, three engineering design activities designed for a middle school mathematics course conducted according to an integrated STEM approach are presented and excerpts from the activity implementations are included. Activities involve a problem that needs to be addressed with its conditions, a research process to find a solution, and testing and control processes that need to be applied to reach what is considered to be the best result. In addition, the activities included inquiries to support conceptual learning about the gains of ratio and proportion. However, one limitation of the activities in this manuscript is that it aims to cover the learning outcomes in the curriculum. At this level, the learning outcomes addressed proportional reasoning more in terms of expressing relationships between multiplicities. For this reason, the mathematical concepts related to the activities were discussed in terms of what they express in real life and how they explain what they explain. As the activities progressed, it was observed that students were more successful in expressing relationships between multiplicities in situations requiring proportional reasoning. In the activities to be designed for future research, the concepts of ratio and proportion can be handled through a process that will enable students to acquire proportional reasoning skills. This process can be planned to address components such as multiplicative relationships, changes between quantities, comparing and interpreting proportional relationships in different contexts.

It was observed that students did not hesitate to use content related to mathematics and science in developing solutions for design. In their investigations for different solutions, they easily used even their newly acquired knowledge in engineering and science. Therefore, it is thought that the use of theoretical math and science content should be encouraged in the content designed for the integration of engineering design into STEM education. For the retention of the acquired knowledge, the activities can be expanded in a way that students can apply this new knowledge to different problems.

The STEM approach is more preferred in science courses and the role of mathematics in integrated STEM activities is ignored (Kristensen et al., 2024). However, it has been recognized that mathematics forms the basis of other STEM disciplines (Maass et al., 2019). Mathematics can provide a tool, method or context for science, technology and engineering. In some studies, it has been stated that mathematical modeling should be central to address mathematics in STEM with real-world problems (Just & Siller, 2022). Although the activities in this manuscript do not directly apply mathematical modeling processes, scenarios are designed in which students can work on models to use mathematical knowledge. Considering the contribution of mathematical modeling processes to students' problem solving and interdisciplinary thinking skills, it is suggested that these scenarios be expanded to allow students to be more involved in the process of creating their own models.

It was observed that the activities increased students' active participation in the lesson and students were willing to participate in the activities. At the same time, it was concluded that students were more likely to participate in the third activity where active participation opportunities were more intense. In the literature, it is stated that in order for STEM education contents to create interest and curiosity in students, they should be free in designing and these contents should be planned in a way suitable for them to use their creativity (Abdurrahman et al., 2023; Hathcock et al., 2015). Based on this result, it is thought that in future research, a design process in which mathematical concepts and ideas are used directly without the constraints of subject acquisitions can be created.

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APPENDIX

Appendix 1. Two Gear Activity Worksheet

2 Gear Worksheet

Number of driven gear teeth	Number of driving gear teeth	Gear ratio

Gear Combination	Calculated Output speed	Measured Output speed	Gear ratio
40-8			
8-40			
40-24			
24-40			
8-24			
24-8			

The combination you chose for the acceleration gear:
The combination you choose for the deceleration gear:
The combination you choose for a powerful start:

In a system where the input speed is 5 and the output speed is 10, how many revolutions does the output lever turn if you turn the input lever 1 revolution?

In a gear system where the number of teeth of the rotating gear is 8, how many teeth should the rotating gear have so that the output speed is 2 while the input speed is 6?

Appendix 2. Hydraulic Leverage Activity Worksheet

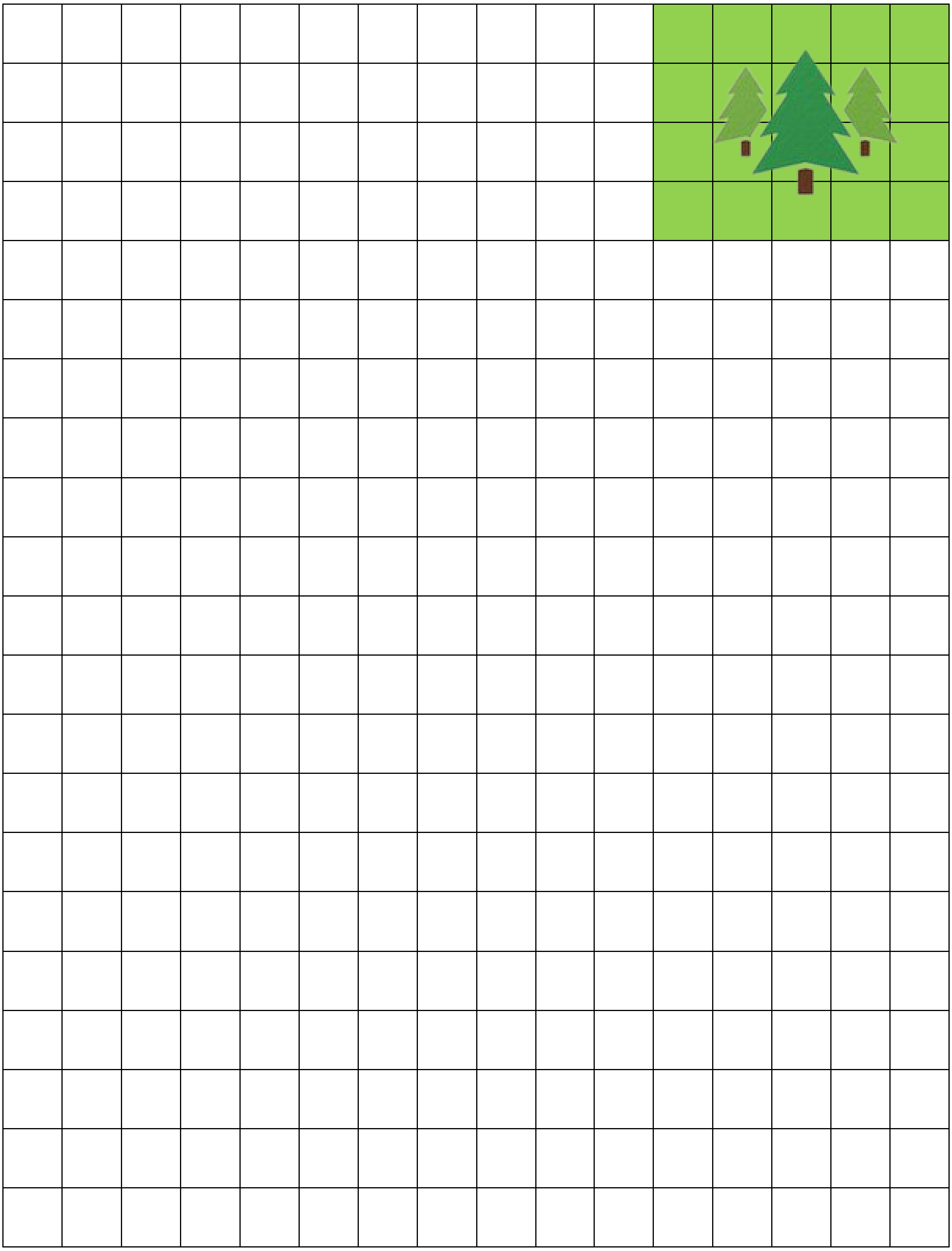
Hydraulic Leverage Activity Worksheet

1. Syringe Cross-Sectional Area	2. Syringe Cross-Sectional Area	1. Weight	2. Weight

Write down the fixed weight you will use in your design, the syringe it will be placed in and the syringes to which force is applied:

Explain the reasons for your choice:

Appendix 3. Life Center Activity Design Sheet



Appendix 4. Life Center Activity Worksheet

Life Center Activity Worksheet

We are looking for a team to build a residential center on a vacant lot to be developed for a new housing development. The team with the best design according to the given conditions will win and the residential center will be built according to that team's design. Determine the structures you would like to have in your residential center and transfer your design to the plan. Remember that you make your choice based on the time and cost to implement your design.

- **What are the structures that should be in your life center? Write below (hospital, school, roads... etc.)**

- **There is a forested area within the boundaries of the area that you will turn into a living center. It is known that this forest area is 200000 m2. Accordingly, calculate the scale you will use in the plan on the design paper you have been given a draft of.**
  Scale:

- **Write the names of the buildings you used in your design, their dimensions on the map and their actual dimensions. (For example; School Area on Map: 25 m2 Real Area: 12500 m2**

- **Calculate and write down the total cost and time required to realize your design for each building.**

Appendix 5. Cost Sheet for the Life Center Activity

Life Center Cost Sheet

	m² cost	time required for m² (with 1 worker)
Building (1 floor)	200 units	25 units time
Grass field	50 units	10 units time
Woodland	100 units	20 units time
Stony road	25 units	5 units time
Asphalt road	75 units	15 units time

Cost of each worker: 30 units

Note 1: m2 cost data is only valid for the materials of the structure. Do not forget to include the labor cost for the total cost.
Note 2: You can employ a different number of workers for the construction of each structure.
Note3: You need to calculate the construction time of the structures in such a way that after one is completed, the other one starts.