# The Effect of Geometry-Based Brain Teasers on the Geometric Thinking Levels and Spatial Visualization Skills of Middle School Students

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In this study, the effects of geometric-based brain teasers on the geometric thinking levels and spatial visualization skills of middle school students were examined. A single-group pre-test-post-test experimental design, which is a quantitative research method, was used in the study. The study sample comprised 20 middle school students attending a public school. Within the scope of the study, the experimental group played the familiar games, such as Tangram, Motif, Equilibrio, and Penta Block, as well as the Geoboard-GT and Geoboard-SV games designed by the researchers and applied to the geoboard for a total of six weeks. The Turkish adaptation of the Van Hiele Geometry Test (developed by Usiskin, 1982, and adapted by Duatepe, 2000) and the Spatial Visualization Test developed by Olkun (2003b) were used as data-collection instruments. In the data analysis, the Wilcoxon Signed-Rank test, a nonparametric test, was used because the group's pre-test and post-test scores did not follow a normal distribution. The findings revealed that students' Van Hiele Geometry post-test scores increased significantly compared with their pre-test scores across specific levels, total scores, and assigned levels. A significant difference was found between the pre-test and post-test scores on the spatial visualization test, favoring the post-test. In addition, although a significant difference was observed in favor of post-test scores for the spatial and area measurement sub-dimensions of the spatial visualization test, no significant difference was observed for the numerical-spatial and mental rotation dimensions.

**Keywords:** Geometry; brain teasers; geometric thinking level; spatial visualization; secondary school students.

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## Introduction

Geometry is a subfield of mathematics that plays a significant role in the concretization of mathematical concepts, encompassing many of the shapes and structures encountered in daily life. Therefore, the primary objective of geometry instruction in schools should be to enable students to think and reason by establishing connections between mathematical domains, while also fostering skills such as logical consistency in their findings (Özdemir & Çekirdekçi, 2022). In geometric thinking, establishing relationships between geometric shapes and concepts is paramount for solving geometry problems (Van de Walle, Karp, & Bay-Williams, 2010).

Geometry instruction should prioritize the development of students' skills in comprehending the properties of two- and three-dimensional objects, as well as their spatial visualization and reasoning capabilities (National Council of Teachers of Mathematics [NCTM], 2000). Spatial visualization and geometric thinking are not only significant in geometry education but also represent fundamental human skills essential for daily life. Furthermore, these skills are required in various fields and professions, including engineering, design, graphics, science, chemistry, and the arts.

Although not all individuals think about geometric concepts in the same way, everyone possesses the ability to think and reason in geometric contexts, and this ability can be improved. The studies of Pierre van Hiele and Dina van Hiele-Geldof provide insights into the differences in geometric thinking and how these differences arise (Van de Walle et al., 2010). The work of this Dutch couple suggests that students' difficulties in understanding geometric concepts stem from a failure to account for their levels of geometric thinking (Usiskin, 1982).

Although some argue that spatial reasoning is innate, research shows that this skill can be developed starting from early childhood (Pavlovičová, Bočková, & Laššová, 2022). In this context, it can be stated that just as families influence the development of a child's spatial skills, teachers also play an important role in developing students' spatial skills during their school years (Pavlovičová et al., 2022). Therefore, activities aimed at developing spatial skills should be integrated into the educational environment by teachers.

There are several recommendations in the literature regarding activities to improve spatial skills. In line with Van Hiele's (1999) statement that 'geometry begins with play,' geometry lessons should be designed to allow students to play with geometric concepts and shapes (Duatepe Paksu, 2023). Simultaneously, games and activities that enable students to encounter geometric objects of different sizes and positions and prompt them to reason about the properties of these objects should be selected (Duatepe Paksu, 2023). In particular, the development of geometric thinking can be supported through games and activities associated with geometric shapes and patterns (Van Hiele, 1999). Furthermore, studies indicate that geometry-based games and activities, conducted at both individual and group levels, improve spatial visualization (Osberg, 1997).

According to Clements and Battista (1992), there is a significant association between students' spatial skills and their levels of geometric thinking. For example, an individual at Van Hiele Level 1 judges shapes based on their appearance. However, for individuals at Level 2 and above, spatial visualization will be supported by their knowledge of the object's properties. The ability to mentally visualize transformations applied to objects is directly related to the structural properties of the object (Clements & Battista, 1992). Therefore, students at different geometric levels exhibit differences in their spatial skills. In this context, this study examines the effect of geometry-based brain teasers on geometric thinking levels and spatial visualization skills. In this regard, the Van Hiele Geometric Thinking Model and the concepts of spatial visualization, which constitute the theoretical framework of the research, will first be explained.

## Van Hiele Geometric Thinking Model

The Van Hiele Geometric Thinking Model was developed by Dina van Hiele-Geldof and Pierre van Hiele in the 1950s. This model seeks to explain how geometry is perceived at different levels. The most distinctive feature of this model is its five-level hierarchical structure that defines the thinking processes employed in geometric contexts. These levels describe not how much knowledge one possesses, but the nature of one’s geometric thinking (Duatepe Paksu, 2023; Van de Walle et al., 2010).

**First Level (The Visual Level):** This is the foundational level and begins with non-verbal thinking. At this visual thinking level, shapes are judged according to their appearance. Students at this level might state, "This is a rectangle because it looks like a box" (Van Hiele, 1999). At this stage, students evaluate, recognize, and name geometric shapes. For instance, they evaluate a parallelogram without considering its side and angle properties. At this level, the position, orientation, and appearance of geometric shapes take precedence.

For example, when a square is rotated by 45 degrees, students at this level will no longer consider it a square; instead, it will be labeled as a different shape, such as a kite or a rhombus (Van de Walle et al., 2010). When working with students at this level, emphasis should be placed on shapes that they can observe, decompose, or construct. Activities aimed at exploring the similarities and differences of these shapes should be conducted (Duatepe Paksu, 2023).

**Second Level (The Descriptive Level):** At the descriptive level, students can understand that shapes possess specific properties. A shape is no longer a square because it "looks like" a square, but rather because it possesses specific properties (Van Hiele, 1999). For students at this level, the properties of geometric objects take precedence over their visual appearance (Duatepe Paksu, 2023). Students know that the properties that define a rectangle include four right angles and opposite sides that are parallel and equal in length (Van de Walle et al., 2010).

At this level, attributes such as the size and orientation of geometric shapes begin to fade into the background for students. However, while students at this level can list all the properties of squares, rectangles, and parallelograms, they may not yet perceive that these shapes are subsets of one another. Therefore, they cannot infer that a square is also a rectangle (Van de Walle et al., 2010). Furthermore, students at this level cannot yet grasp the relationships among the properties of a shape. Consequently, when required to define a geometric shape, they cannot provide an "economical" definition; instead, they list all the properties they know about the shape (Duatepe Paksu, 2023).

**Third Level (The Theoretical Level / The Informal Deduction Level):** At this level, students begin to establish relationships between the properties of geometric shapes. For example, they can engage in reasoning such as: "A rectangle is a quadrilateral with four right angles. A square is a quadrilateral with all angles right, and all sides equal. In this case, a square must be a type of rectangle" (Van de Walle et al., 2010).

As the reasoning ability improves, they can define shapes in an "economical" manner. At the same time, they can conclude that a shape can have multiple valid definitions. They can compare two different definitions for a rhombus, such as "a quadrilateral with all sides equal" or "a quadrilateral with an interior angle sum of $360^o$ whose opposite angles and sides are equal" (Duatepe Paksu, 2023). At the level of informal deduction, the properties of shapes are logically ordered. They are derived from one another; one property causes or is deduced from another (Van Hiele, 1999). Students at this level can make intuitive deductions, but they cannot yet perform formal deductive logical proofs (Van de Walle et al., 2010).

**Fourth Level (Formal Logic):** Students at this level prove theorems deductively and establish reciprocal relationships between theorems (Fuys et al., 1988). While Van Hiele characterized the third level as the essence of geometry, he delineated the fourth level as the essence of mathematics (cited in Duatepe Paksu, 2023). This is due to the fact that students at this level are capable of making logical inferences to prove other theorems. For example, they can prove that the sum of the interior angles of an n-sided polygon is (n-2) x$180^o$ (Duatepe Paksu, 2023)

**Fifth Level (The nature of logical laws):** Students at this level can construct theorems within different axiomatic systems and analyze and compare these systems (Fuys et al., 1988). Consequently, individuals who attain this level of expertise are capable of working with non-Euclidean geometries, constructing axioms and theorems, and performing proofs (Duatepe Paksu, 2023). This level is generally reached by those studying geometry at the university level as a branch of mathematical science (Van de Walle et al., 2010).

The Van Hiele couple argues that progression between geometric thinking levels is possible through appropriate, teacher-guided activities (Duatepe Paksu, 2023). Instruction aimed at supporting development from one level to the next should include activities that begin with an exploratory phase, gradually build concepts and relevant terminology, and help students integrate what they have learned with their prior knowledge. Simultaneously, the development of geometric thinking can be facilitated through enjoyable games and activities such as pattern blocks, mosaic puzzles, and Tangram (Van Hiele, 1999).

## Spatial Visualization

Spatial visualization is the ability to understand the outcomes of movements—such as rotation, translation, reflection, and folding—of 2D and 3D objects by mentally imagining these movements. In other words, it is the ability to visualize and manipulate geometric objects and shapes in the mind (Kayhan Kırmaç, 2023). Additionally, spatial visualization involves the mental manipulation and integration of stimuli composed of multiple or moving parts (Olkun, 2003a). Clements (2003) defines spatial visualization as the ability to understand and perform imagined movements of 2D and 3D objects.

According to the NCTM (2000) standards, students at the primary education level should be able to display and draw the views of objects from different perspectives. By utilizing their spatial visualization and visual memory skills, they should also be able to place geometric shapes and objects in their environment while establishing connections between these objects and their surroundings.

Clements (2003) contends that the enhancement of spatial visualization skills can be achieved through the implementation of manipulative activities involving geometric shapes, such as tangrams and pattern blocks. Research indicates that activities requiring hand-eye coordination contribute to the development of spatial skills (Sorby, 1999). Usiskin (1987, as cited in Clements & Battista, 1992) addressed the four components of geometry within the framework of spatial skills and defined them as follows:

- Visualization, drawing, and construction of shapes.
- Examination of the spatial aspects of the physical world.
- Use of geometry as a tool for representing non-visual mathematical concepts and relationships.
- Representation of geometry as a formal mathematical system.

The first three of these dimensions necessitate the use of spatial reasoning.

**Research Problem**

It is argued that geometry instruction is not merely a process of memorizing definitions or applying formulas; rather, its purpose is to equip students with the skills necessary to make sense of the physical world around them (NCTM, 2000). Van Hiele’s geometric thinking model emphasizes that students must engage in concrete experiences to reach higher-order thinking skills (Van de Walle et al., 2010). Furthermore, Van Hiele (1999) posited that play occupies a significant role in the geometry education of children, and recommended that teachers should have games such as Tangrams and Pattern Blocks readily available because engaging students in games and activities that support their geometric thinking in the classroom will also make them more engaged and successful in mathematics (Van Hiele, 1999).

The primary rationale for this study is to support students' spatial visualization and geometric thinking skills by providing them with concrete experiences through geometry-based brain teasers. In line with this rationale, the research focuses on geometry-based games, and the activities were designed specifically for this purpose using a geoboard.

When selecting these geometry-based games, in addition to widely known games such as Tangram and Penta Block (Katamino), games like Motif and Equilibrio, which are used in tournaments within the scope of the protocol between the Turkish Federation of Mind and

Intelligence Games (TAZOF) and the Ministry of National Education (MEB), were included in the study. A novel aspect of this research is the design of construction activities in the style of brain teasers, enabling students to perform these constructions on a geoboard. The utilization of a geoboard in geometry instruction has been demonstrated to elicit heightened student interest in the lesson, foster effective in-class communication, and promote active learning (Sibiya, 2020).

Consequently, two distinct brain teaser-based activities were meticulously designed to enhance students' geometric thinking levels (Geoboard-GT) and spatial visualization skills (Geoboard-SV) through engagement with the geoboard. In this respect, the study differs from existing literature by examining the effects of these games on both geometric thinking and spatial visualization simultaneously. To investigate the impact of geometry-based brain teasers on the geometric thinking levels and spatial visualization skills of middle school students, this study seeks answers to the following research questions:

1. Is there a statistically significant difference between the pre-test and post-test mean scores of the Van Hiele Geometric Thinking Test for students participating in the geometry-based brain teaser activities?
2. Is there a statistically significant difference between the pre-test and post-test mean scores of the Spatial Visualization Test for students participating in the geometry-based brain teaser activities?

## Method

The present study employed a single-group pre-test-post-test experimental design, a quantitative research method. In this design, the participants' levels are determined by a pre-test before the intervention and a post-test after the intervention. The efficacy of the intervention is determined by assessing the existence of a statistically significant discrepancy between the participants' pre-intervention and post-intervention scores (Baştürk, 2021).

The Van Hiele Geometry Test, developed by Usiskin (1982) and adapted into Turkish by Duatepe (2000), and the Spatial Visualization Test, developed by Olkun (2003b), were used as the data collection tools. The Van Hiele Geometry Test contains five questions for each of the five geometric thinking levels. To determine whether participants have reached a specific level, a criterion of answering 3 or 4 out of 5 questions correctly per level is typically applied (Senk et al., 2022). Usiskin (1982) developed the geometry test to assess high school students' ability to "identify" and "predict" their performance within the Van Hiele theory. In this context, the initial sample during the development phase of the Van Hiele Geometry Test consisted of participants, 96% of whom were in the 14-17 age range (high school level) (Usiskin, 1982). In this study, since the participants were in the 10-13 age range (middle school level), the requirement of answering 3 out of 5 questions correctly per level was used to assign participants to that specific level. In addition, the mathematics curriculum for primary and middle school students progresses only to the third level (Van Hiele, 1986, as cited in Duatepe, 2004); thus, the initial 15 questions, which encompass material up to the third level, were utilized in this study. In the Van Hiele Geometry Test, the correct responses provided by participants were scored as 1 point, while the incorrect responses were assigned a value of 0 points. Since there were five items in each level of the test, the maximum score obtainable per level was 5. Because this study utilized questions from the first three levels of the Van Hiele Geometry Test, the maximum score obtainable on the test was 15. Duatepe (2004) calculated Cronbach's Alpha reliability coefficients for the test as .82 for the first level, .51 for the second level, and .70 for the third level. In the present study, Cronbach's Alpha reliability coefficient for the 15-question portion of the test was calculated as .66. Furthermore, the study utilized a version of the Spatial Visualization Test, originally developed by Olkun (2003b) with 24 items, to which 5 items were later added (Olkun & Altun, 2003). The spatial visualization test consisted of four sub-dimensions: the spatial dimension (9 items), numerical-spatial (7 items), mental rotation (8 items), and area measurement (5 items) (Olkun & Altun, 2003). In the spatial visualization test, correct responses provided by the participants were scored as 1 point, and incorrect responses as 0 points. Proportionate to the number of items in the sub-dimensions, the maximum scores obtainable were 9 for the spatial dimension, 7 for the numerical-spatial dimension, 8 for the mental rotation dimension, and 5 for the area measurement dimension, with a maximum total score of 29 points for the test as a whole. Olkun and Altun (2003) determined the reliability coefficient for this 29-item spatial visualization test to be $\alpha$ = .78 (N=233). In the present study, Cronbach's Alpha reliability coefficient for the spatial visualization test was calculated as $\alpha$ = .86 (N=20).

The population of the study comprised all middle school students in a district center within the Mediterranean region. The sample consisted of 20 students from a middle school located in the same district. The study employed convenience sampling, a type of non-probability sampling method. In this method, the researcher identifies accessible cases with the objective of reducing the time required for data collection (Kılıç, 2013). In this context, the sample was determined as the students attending the "Mind and Intelligence Games" workshop course at the secondary school where the first researcher is employed under the Ministry of National Education (MEB).

**The Implementation of the Activity**

This study was conducted during the 2024-2025 academic year at a public middle school located in a district in the Mediterranean region. The activities determined within the scope of the study were carried out with 20 students enrolled in the "Mind and Intelligence Games" workshop course, under the guidance of the first author. The research group consisted of 8 female (40%) and 12 male (60%) students. Regarding the grade levels, 7 students (35%) were in the 5th-grade, 4 (20%) in the 6th-grade, 3 (15%) in the 7th-grade, and 6 (30%) in the 8th-grade. Prior to the activities, the Van Hiele Geometry Test and the Spatial Visualization Test were administered as pre-tests. Subsequently, a geometry-based brain teaser intervention was implemented, spanning a total of 12 lesson hours over a 6-week period. Upon completion of the intervention, post-tests were administered. The implementation process and the specific brain teaser activities conducted during this period are presented in Table 1.

**Table 1**

*Implementation Process*

| | |
|---|---|
| Pre-test | |
| Week 1 | Tangram + Mathigon |
| Week 2 | Motif |
| Week 3 | Equilibrio |
| Week 4 | Penta Block |
| Week 5 | Geoboard-GT |
| Week 6 | Geoboard-SV |
| Post-test | |

In the selection of the brain teasers for this study, meticulous care was exercised to ensure that the games directly supported students' spatial visualization and geometric thinking skills. The common feature of the selected games was their capacity to elicit skills such as mental rotation and translation in students. Furthermore, games aimed at developing students' thinking skills in both the two-dimensional plane (e.g., Tangram, Motif) and the three-dimensional space (e.g., Equilibrio) were selected. Strategic brain teasers commonly played by students in middle schools, such as Mangala, Reversi, Pentago, and Kulami, were not included in this study. This is due to the fact that these games are predominantly characterized by trial-and-error, problem-solving, logic, and strategy development skills (Kel & Kul, 2021). Given the study's emphasis on geometric concepts and spatial relationships, geometry-based games were selected for analysis. In this context, in addition to ready-made geometry-based brain teasers (e.g., Tangram, Motif, Equilibrio, Penta Block), geoboard activities structured by the researchers were also developed as part of this study. The geoboard activities were designed to support students' Geometric Thinking (GT) and Spatial Visualization (SV) skills. The tasks designed to assess geometric thinking (GT) in students required them to engage in constructions that utilize the geometric properties of shapes, including diagonal properties, parallelism, perpendicularity, and side lengths. Conversely, SV tasks were meticulously designed to facilitate students' mental visualization of the post-transformation states of shapes and to enable the mental manipulation of area-perimeter relationships. When planning the geometry-based brain teaser activities for the implementation process, the five-stage activity sequence proposed by Van Hiele (1999, p. 316) to encourage the transition between geometric thinking levels was utilized. The activity stages were as follows:

**Inquiry:** In this phase, the teacher asks various questions to draw attention to the activity and guides students to explore specific structures using instructional materials.

**Direct Orientation:** In this phase, the tasks are assigned to help students perceive geometric properties and relationships through games.

**Explicitation:** In this phase, the teacher introduces key terms and concepts related to geometry and fosters a collaborative learning environment through class discussions.

**Free Orientation:** In this phase, the teacher presents tasks that can be completed in various ways, enabling students to become more proficient by utilizing their prior knowledge.

**Integration:** In this phase, students are given the opportunity to consolidate what they have learned by creating their own activities (Van Hiele, 1999).

The six geometry-based brain teasers selected to enhance students' spatial visualization and geometric thinking skills were planned according to the five-stage instructional process defined above, with the aim of positively influencing their geometric thinking levels. Although students worked individually during these phases, they were given the opportunity to share their solutions with their peers to maintain a high level of classroom interaction. Students presented their solutions for the Tangram game using the Mathigon application on the smartboard. Similarly, they shared their solutions for the geoboard games by opening the digital versions on the smartboard. For the Motif, Equilibrio, and Penta Block games, for which digital versions were unavailable, students demonstrated their solutions using physical materials. The step-by-step activity process concerning the brain teasers conducted with the research group was as follows:

**Week 1: Tangram + Mathigon (Duration: 40 min + 40 min)**

During the implementation process, 7-piece classical Tangram sets and the Mathigon digital platform were used simultaneously. At the beginning of the lesson, worksheets and 7-piece classical Tangram sets were distributed to the students. The step-by-step activity process proceeded as follows:

***Inquiry:*** The question, "What can we do with these pieces?" was posed to the students. Students were encouraged to respond to this question by utilizing their imagination and playing with the pieces. At this stage, it was observed that students provided responses such as, "A roof for a house can be made," "When the triangles are combined, they form a cat's ear," and "When we combine them all, they form a square."

***Direct Orientation:*** Students were asked to identify all the shapes that could be formed from two pieces. Subsequently, students were first asked to form a square using 2 pieces, followed by forming a square using 4 pieces. At this stage, it was observed that students explained their solution processes with comments such as, "It becomes a square when the long sides of two small triangles are combined," and "When I placed the triangle next to the parallelogram this way, the sides did not match, so I rotated the triangle." These statements indicate that students began to notice the side and angle properties of the shapes and were analyzing the relationships between the pieces. It was observed that as students created these shapes, they began to establish visual connections with the angles and matching sides of the shapes they utilized.

***Explicitation:*** The teacher provided information regarding the angle and side properties of triangles, squares, and parallelograms using the Tangram pieces the students had manipulated.

***Free Orientation:*** Mathigon software was opened, and student responses were discussed on the smartboard. Students were asked to form the different shapes found on the worksheet (see Figure 1).

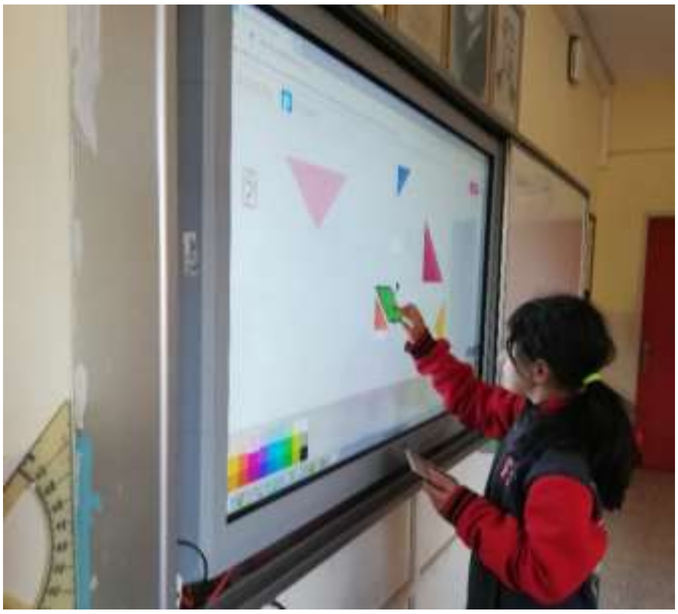

**Figure 1.**

*In-class Tangram + Mathigon Activity*

***Integration:*** Students were asked to create any geometric shape, animal, object, etc., using the Tangram pieces available to them.

The primary objective of this activity was to enable students to discover side and angle properties by relating Tangram pieces to one another. In addition, the aim was to improve spatial visualization skills by creating new shapes through transformations such as rotation and translation.

**Week 2: Motif (Duration: 40 min + 40 min)**

In this activity, the game Motif was played by applying transformations, such as rotation and translation, to the pieces and combining them. Given the game's structural features, the aim was for students to understand symmetry, translation, and rotation, thereby improving their spatial visualization and geometric thinking skills. The activity process proceeded through the following stages:

***Inquiry:*** The teacher introduced the game Motif to the classroom and asked the students to discuss what could be done with the game pieces.

***Direct Orientation:*** Students were asked to choose a game card of their preference and create the same Motif using the game pieces.

***Explication:*** The teacher explained the concept of symmetry by discussing the symmetric shapes formed on the different Motifs created by the students. The teacher posed questions such as, "What can you say about the parts we obtain if we divide this Motif you created exactly in half?" Students responded, "The same shape is formed on both sides." Through these statements, the teacher explained the concepts of the line of symmetry and reflection.

***Free Orientation:*** Students were asked to select a card from the game cards that they believed possessed the most lines of symmetry and construct it on the game board. At this stage, students responded with comments such as "I chose this one because it has both horizontal and vertical lines of symmetry," and "The one I chose also has a diagonal line of symmetry." These discussions demonstrate that the students internalized the concepts of symmetry and lines of symmetry.

***Integration:*** Students were asked to create a Motif containing symmetric patterns independently of the existing game cards using the game pieces. A change was made to the game rules here. According to the standard game rules, the player who constructs the pattern on the task card the fastest wins the game. In this new version, the student who creates the motif with the greatest number of symmetry lines wins. The purpose of this activity is to develop students' spatial skills during the Motif creation stage and to enable them to grasp concepts such as symmetry by noticing geometric shapes in the Motifs they create.

**Week 3: Penta Block (Duration: 40 min + 40 min)**

In this activity, using the Penta Block game, which uses 12 pieces, each consisting of 5-unit squares, the goal was to support students' spatial visualization skills, such as conservation of area, figure-ground relationships, and transformation geometry. The activity process proceeded as follows:

***Inquiry:*** The teacher introduced the Penta Block game to the classroom and asked students to discuss how the shape blocks could be used. Students responded with comments such as, "We can play Tetris," or "We can fill a rectangular area." However, students did not initially realize that each of these pieces had an area of 5-unit squares. When the teacher asked, "What do you think about the areas covered by the pieces?" it was observed that some students realized they had equal areas.

***Direct Orientation:*** Students were asked to place the game divider at position number 4 on the game board, select any 4 Penta Block pieces, and construct the area. The same task was also conducted for "Penta 5." The aim here was to enable students to select shapes according to the purpose, rotate them, and use them as desired to obtain the target area.

***Explication:*** Placing pieces to fill an area in games provides experiences related to the area. Through direct comparison, students can demonstrate that some pieces occupy more space than others or discover relationships such as one piece being half the size of another (Van Hiele, 1999). The teacher provided an explanation on how to calculate the area of different geometric shapes in the game in terms of square units and how to compare the areas of these shapes. In classroom discussions, students responded by saying, "It took 4 pieces to cover this area; each is a 5-unit square, so there is a total space of 20-unit squares. My friend covered the 20-unit square area using different pieces." This situation demonstrates that students have internalized the concept of area measurement using non-standard units.

***Free Orientation:*** Students were asked to complete the tasks in the game booklet. An interactive environment was provided while performing these tasks, and students who completed the task were asked to demonstrate it to those who could not. At this stage, it was observed that students gave instructions such as "Rotate 90°, flip, slide." This indicates that it strengthened the students' skills in expressing spatial relationships.

***Integration:*** Finally, students were asked to fill any area on the game board using shapes of their choice, independent of the task cards. At this stage, students gave responses such as, "I can fill the area better when I use the 3 pieces here," or "I cannot fill the area when I use this piece." This demonstrates that they manage the problem-solving process by utilizing their spatial skills in terms of part-whole relationships.

**Week 4: Equilibrio (Duration: 40 min + 40 min)**

In this activity, the aim was to enhance students' spatial visualization skills by transforming 2D visuals into 3D structures, and to develop their geometric thinking skills by analyzing the surface, edge, and vertex properties of geometric solids. The activity process is summarized below:

***Inquiry:*** The teacher introduced the Equilibrio game to the classroom and asked students to discuss which geometric structures the game materials were composed of. Students in lower grade levels responded with descriptions such as "It looks like a box," "It looks like a battery," or "It looks like a roof," while students in higher grade levels responded with terms such as "A cube because all its faces are squares," or "This is a cylinder, the other is a rectangular prism." Since the research group included students from different grade levels of the secondary school, the aim here was to ensure that students at lower grade levels recognized different geometric shapes. Through classroom discussion, it was observed that students in the lower grades recognized three-dimensional objects.

***Direct Orientation:*** Students were asked to build the tallest possible structure by stacking the geometric structures in the game. At this stage, expressions such as "If I lay the cylinder on its side, it rolls; I cannot place anything on top of it. Therefore, I must place the cylinder upright," and "I cannot place another object on the pointed tip of the triangular prism, so we must place it on top," were observed among students. It was noted that at this stage, students used their own terminology rather than “surface," "edge," or "vertex."

***Explication:*** Discussions were held regarding which surfaces were in contact, and whether vertex-surface or edge-surface contact in the structures they built. The teacher provided explanations such as, "We call the pointed ends 'vertices,' the flat areas 'surfaces,' and the line where two surfaces meet an 'edge'." The aim was to teach students the concepts of vertex, surface, and edge through the game’s geometric structures.

***Free Orientation:*** In the next part, students were asked to complete the tasks in the game booklet as quickly as possible. While performing these tasks, the structures they had difficulty building and the reasons behind these difficulties were discussed.

***Integration:*** Students were asked to build a structure of their choice using all available game pieces, independent of the task cards, while considering the vertex, edge, and surface properties of the geometric shapes. In the standard rules of Equilibrio, the player who builds the structure in the task booklet the fastest without knocking it over wins. In the game phase of this study, however, the rules were modified to ensure that students gained an understanding of concepts such as the surfaces, vertices, and edges of 3D geometric solids.

**Week 5: Geoboard-GT (Duration: 40 min + 40 min)**

In this activity, students were required to perform constructions on the geoboard based on questions designed by the researchers. The student approaches observed during this process appeared to reflect their development in geometric thinking. The step-by-step activity process proceeded as follows:

***Inquiry:*** Geoboards and enough rubber bands were brought to the classroom by the teacher so that each student had one. Students were asked to demonstrate what they could do with these materials. At this stage, it was observed that students created forms resembling objects such as houses, stars, and letters rather than geometric shapes. Students responded, such as "It became an hourglass when I crossed the rubber bands."

***Direct Orientation:*** Students were asked to construct one geometric shape each on the geoboard.

***Explication:*** Concepts such as vertex, edge, diagonal, polygon, concave, and convex were introduced by discussing the geometric shapes the students had constructed.

***Free Orientation:*** Students were asked to perform the brain teaser style constructions designed by the researchers on the geoboard using rubber bands. The requested constructions were directed to the students sequentially, and all students worked individually. Subsequent to each inquiry, students who successfully arrived at a solution were prompted to articulate their methodologies on the digital geoboard, which was displayed on the smartboard. The constructions requested from students during the activity phase were designed to enhance geometric thinking. The following constructions were requested:

- Construct a polygon with no diagonals.
- Construct a polygon with two diagonals.
- Construct a quadrilateral with opposite sides parallel and of equal length.
- Construct a rectangle whose diagonals bisect each other at right angles.
- Construct a quadrilateral whose diagonals are perpendicular to each other, but whose interior angles are not right angles.
- Construct a parallelogram with equal diagonal lengths.

At this stage, while it was observed that some students directly created a triangle when constructing a polygon without diagonals. In contrast, other students were observed to create a quadrilateral first and then proceed to reduce the number of sides. In the construction of a rectangle whose diagonals bisect each other at right angles, it was observed that the majority of students initially attempted to reach the solution through the construction of a rectangle with two long and two short sides. After a few attempts, some students responded, "I did it, but it became a square." In response to the inquiry posed by the teacher, "Is a square equivalent to a rectangle?" several students posited, "Yes, it is, given that all of its angles are right angles." In the task of constructing a quadrilateral whose diagonals are perpendicular, but whose interior angles are not right angles, it was observed that students generally constructed squares whose sides were not parallel to the edges of the geoboard. In response to the instructor's inquiry regarding the potential of this construction to be a square, some students successfully constructed a rhombus that was not congruent with a square. In the construction of a parallelogram with equal diagonal lengths, it was observed that some students constructed a rectangle, while others constructed a square. It was determined that while students required a greater investment of time to complete their initial constructions, they were able to complete the constructions in the final part more quickly. This finding suggests that they initiated the process of conceptualizing the relationships between geometric shapes.

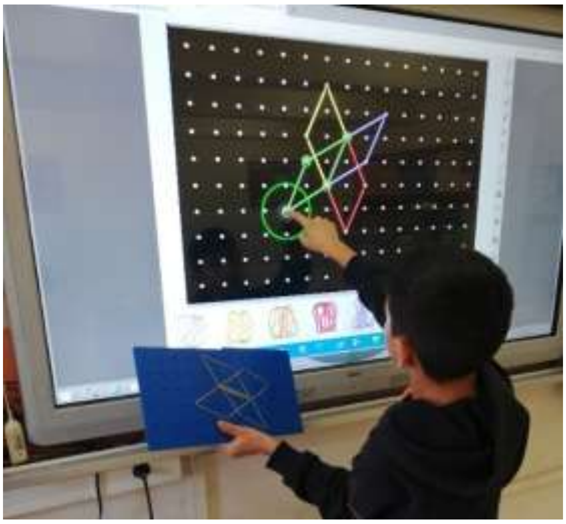

**Figure 2.**

*In-Class Geoboard-GT Activity*

***Integration:*** Subsequent to the construction tasks previously delineated, it was observed that students generally began to develop an understanding of the angle, side, and diagonal properties of triangles and quadrilaterals, as well as their hierarchical relationships. Finally, in the integration phase, they were instructed to complete the following tasks:

- Construct four different rhombi sharing one common edge.
- Construct a rhombus whose diagonals are not parallel to the edges of the geoboard.

In this stage, tasks of a more complex nature were selected. For instance, the task of constructing a rhombus whose diagonals are not parallel to the edges of the geoboard was designed to ensure that students comprehend that geometric properties are independent of orientation. It was observed that students struggled with this task initially; however, after completing the construction in the first task, they were observed to construct the rhombus with diagonals not parallel to the geoboard edges more quickly. It was noted that while performing the second construction, some students constructed the diagonals of the rhombus first. This finding suggests that students have developed an understanding of the preservation of geometric properties, such as side, angle, and diagonal measurements, even in the context of shape rotation.

**Week 6: Geoboard-SV (Duration: 40 min + 40 min)**

Geoboard-SV is a brain teaser designed by researchers to enable students to perform constructions and calculations on a geoboard. Each student worked individually on their own geoboard. Students who carried out the activities in the game shared their solutions with their peers using the digital geoboard displayed on the smartboard. The questions in this game were primarily aimed at improving students' spatial visualization skills. In this regard, the aim was for students to discover the position-orientation relationships of geometric shapes and area-perimeter relationships through the constructions they made on the geoboard. The step-by-step activity process proceeded as follows:

***Inquiry:*** First, students were asked how they would find the area of the board, accepting the space between two nails as one unit. Second, they were asked how the image of the board would appear when rotated 90° (a quarter turn). In response to the inquiry regarding the perceived absence of observable change upon turning the board, some students posited that a shift in the nails' position was indeed occurring, albeit imperceptibly. Specifically, they contended that the nails on the right side had moved downwards. This phenomenon exemplifies the students' recognition that an object's identity remains constant, irrespective of its physical location.

***Direct Orientation:*** The students were instructed on the use of the geoboard as follows:

- Construct a rectangle and show its image when rotated 90°.
- Construct a square with a side length of 4 units and find its area in square units.

At this stage, some students asked in which direction (clockwise or counterclockwise) the 90° rotation occurred in the first task. In the classroom discussion, it was observed that students determined the direction of rotation did not make a difference for this task. This finding suggests that students possess the capacity to mentally manipulate objects. In the context of a square with a side length of 4 units, numerous students responded by enumerating the unit squares contained within the square. This finding suggests that the subjects' spatial area measurement skills have been engaged.

***Explicitation:*** The teacher explained rotational movement and how to find area using unit squares by providing examples through geometric shapes.

***Free Orientation:*** Students were instructed to perform the following constructions:

- Construct a polygon that looks the same as its initial image when rotated 90°.
- Construct two different polygons that look the same as their initial image when rotated 180°.
- Construct a polygon that looks the same as its initial image when rotated to 60.
- Construct three different triangles and find their areas in square units without using the area formula for triangles.

At this stage, while constructing a polygon that maintains the same appearance as its initial image when rotated 90°, some students responded, "I attempted to use a rectangle, but it did not yield the intended result. However, I was able to achieve the desired outcome by employing a square." For the construction of two different polygons that look the same as their initial image when rotated 180°, they were observed using phrases such as, "This time, both a square and a rectangle work." It was observed that students struggled most with constructing a polygon that looks the same as its initial image when rotated to 60. A few students performed this construction and explained the solution to the class. It was observed that at this stage, students began to use their mental rotation skills relative to the center of the shape through trial and error. In the task of calculating the area by constructing three different triangles, it was observed that students used phrases such as, "I counted the whole squares, then combined the half-squares." This finding indicates that students initiated the process of calculating area on unit squares through the mental partitioning and combination of shapes.

***Integration:*** In order to establish a connection between area and perimeter, students were instructed to perform the following constructions:

- Construct three different rectangles with the same perimeter. Compare the areas of the rectangles.
- Construct three different rectangles with the same area. Compare the perimeters of the rectangles.

At this stage, comparative tasks were provided to change the students' perception that "a larger area means a larger perimeter." In the classroom discussions following these tasks, it was observed that students gained a more profound understanding of the concepts of area and perimeter through spatial reasoning, recognizing that these concepts are independent variables.

**Data Analysis**

The SPSS software package was used to analyze the data collected in the study. A significance level of .05 was considered for the test results. Descriptive statistics were used to determine the pre-test and post-test mean scores of the students obtained from the spatial visualization test and the geometric thinking test. Subsequently, the normality of the distribution of the data was tested to determine the appropriate analysis method to be employed.

For the normality analysis of the participants' pre-test and post-test scores, the results of the Shapiro-Wilk test were considered, as it is recommended for small samples where the number of participants is below 50 (Köklü, Büyüköztürk, & Çokluk, 2022). The normality analysis results of the Van Hiele Geometry Test are presented in Table 2.

**Table 2**

*Shapiro-Wilk Normality Analysis Results for the Van Hiele Geometry Test*

| | **Statistic W** | **Degrees of Freedom** | **p** |
|---|---|---|---|
| **Pre-test** | .793 | 20 | .001 |
| **Post-test** | .803 | 20 | .001 |

*p<.001

As seen in Table 2, it was observed that the significance value for the pre-test and post-test scores of the Van Hiele Geometry Test was .001. Since the significance value was below .05 (p<.05), the group's pre-test and post-test scores were not normally distributed. Therefore, the Wilcoxon Signed Ranks Test, a non-parametric test, was employed for the inferential analysis of the difference between the participants' pre-test and post-test scores.

**Table 3**

*Shapiro-Wilk Normality Analysis Results for the Spatial Visualization Test and its Sub-components*

| | **Statistic W** | **Degrees of Freedom** | **p** |
|---|---|---|---|
| **Pre-test Spatial Dimension** | .772 | 20 | .000* |
| **Pre-test Numerical-Spatial Dimension** | .801 | 20 | .001 |
| **Pre-test Mental Rotation** | .876 | 20 | .015 |
| **Pre-test Area Measurement** | .858 | 20 | .007 |
| **Pre-test Spatial Visualization Test** | .867 | 20 | .010 |
| **Post-test Spatial Dimension** | .728 | 20 | .000* |
| **Post-test Numerical-Spatial Dimension** | .767 | 20 | .000* |
| **Post-test Mental Rotation** | .900 | 20 | .040 |
| **Post-test Area Measurement** | .722 | 20 | .000* |
| **Post-test Spatial Visualization Test** | .817 | 20 | .002 |

*p<.001

In addition, the results of the Shapiro-Wilk normality analysis for the spatial visualisation test (SVT) used in the study and its sub-components, namely Spatial Dimension, Numerical-Spatial Dimension, Mental Rotation, and Area Measurement, are presented in Table 3.

As indicated in Table 3, it was observed that all significance values were below .05 ($p<.05$). In light of the aforementioned results, it was ascertained that the participants' pre-test and post-test scores on the Spatial Visualization Test did not exhibit a normal distribution. Consequently, the significance of the difference between the participants' pre-test and post-test scores for the Spatial Visualization Test and its sub-components was analyzed using the Wilcoxon Signed Ranks Test, a non-parametric test.

## Findings

### Findings Regarding the Impact of Activities on Van Hiele Geometric Thinking Levels

To address the first research problem, "Is there a statistically significant difference between the pre-test and post-test mean scores of the Van Hiele Geometric Thinking Test for students participating in geometry-based brain teaser activities?" the group's Van Hiele Geometric Thinking pre-test and post-test mean scores were first analyzed descriptively, and the results are presented in Table 4.

**Table 4**

*Van Hiele Geometry Test Pre-Test - Post-Test Means of Students Participating in the Activity*

| | N | Mean of Level 1 Questions ($D_1\bar{x}$) | Mean of Level 2 Questions ($D_2\bar{x}$) | Mean of Level 3 Questions ($D_3\bar{x}$) | Mean of Total Score ($G\bar{x}$) | Mean of Assigned Level ($D_T\bar{x}$) |
|---|---|---|---|---|---|---|
| **Pre-Test** | 20 | 3.40 | 1.65 | 0.95 | 6.0 | 1.0 |
| **Post-Test** | 20 | 4.25 | 2.5 | 2.35 | 9.1 | 1.8 |

When the pre-test results of the Van Hiele Geometry Test assessed and evaluated descriptively, it was determined that the participants scored between a minimum of 1 and a maximum of 5 on the first-level questions, between 0 and 3 on the second-level questions, and between 0 and 2 on the third-level questions, with a minimum of 2 and a maximum of 10 points on the overall test. For the post-test, it was revealed that participants scored between a minimum of 2 and a maximum of 5 on the first-level questions, between 0 and 5 on the second-level questions, and between 0 and 4 on the third-level questions, with a minimum of 4 and a maximum of 14 points on the overall test. Parallel to these results, it was observed that the post-test mean scores of the participants presented in Table 4 ($D_1\bar{x} = 4.25$; $D_2\bar{x} = 2.50$; $D_3\bar{x} = 2.35$; $G\bar{x} = 9.10$) showed an increase compared to the pre-test mean scores ($D_1\bar{x} = 3.40$; $D_2\bar{x} = 1.65$; $D_3\bar{x} = 0.95$; $G\bar{x} = 6$) both across the levels and in the overall test. Furthermore, it was observed that the mean level assigned to the participants was 1 ($D_T\bar{x} = 1$) at the end of the pre-test, and this mean score increased to 1.80 ($D_T\bar{x} = 1.8$)at the end of the post-test. The statistical significance of differences in participants' Van Hiele Geometry Test pre-test and post-test mean scores was assessed using the Wilcoxon Signed Ranks Test, and the results are presented in Table 5.

**Table 5**

*Comparison of Van Hiele Geometry Test Pre-Test and Post-Test Means of Students Participating in the Activity*

| | Level 1 Questions | Level 2 Questions | Level 3 Questions | Total Score | Assigned Level |
|---|---|---|---|---|---|
| **Z** | -3.019 | -2.162 | -2.570 | -3.573 | -2.684 |
| **p** | .003 | .031 | .010 | .000* | .007 |

*p<.001

As indicated in Table 5, it was observed that the significance values for Level 1, Level 2, Level 3, and the overall test, obtained from the Wilcoxon Signed Rank analysis applied to the Van Hiele Geometry Test pre-test and post-test scores, were .003, .031, .010, and .000, respectively. The significance value for the assigned level was 0.007. Since these values were less than .05 (p<.05), there was a statistically significant difference. In other words, the Van Hiele Geometry Test pre-test and post-test scores of participants in the research group differ significantly in levels, overall test performance, and assigned levels.

As the research group consisted of secondary school students, a criterion of at least 3 correct answers for each level was required to assign levels to students in the Van Hiele Geometry Test. Usiskin (1982) states that the criterion for level assignment can be 3 or 4 correct answers. In this regard, the researchers tested whether there was a significant difference between the pre-test and post-test scores of the group when a criterion of 4 correct answers was required for level assignment in the Van Hiele Geometry Test. When the level assignment criterion was set to 4, the group's pre-test mean was $\bar{X} = 0.45$, whereas the post-test mean was $\bar{X} = 1.25$. The significance value of the Wilcoxon Signed Ranks Test, applied to compare the pre-test and post-test means, was found to be p=.003. Since the p-value was less than 0.05, there was a significant difference between the group's pre-test and post-test means when the level assignment criterion in the Van Hiele Geometry Test was 4. These results indicate that the 6-week brain teaser activity carried out with the research group positively influenced the group's geometric thinking levels.

### Findings Regarding the Impact of Activities on Spatial Visualization

To address the second research problem, "Is there a statistically significant difference between the pre-test and post-test mean scores of the spatial visualization test for students participating in geometry-based brain teaser activities?", the research group's spatial visualization test pre-test and post-test scores were analyzed descriptively, and the results are presented in Table 6.

**Table 6**

*Descriptive Statistics for the Spatial Visualization Test and its Sub-components*

| | | N | Min. | Max. | Mean (x̄) |
|---|---|---|---|---|---|
| **Pre-Test** | Spatial Dimension | 20 | 2 | 9 | 7.30 |
| | Numerical-Spatial Dimension | 20 | 2 | 7 | 5.05 |
| | Mental Rotation | 20 | 1 | 8 | 5.05 |
| | Area Measurement | 20 | 0 | 5 | 3.30 |
| | Spatial Visualization Test (Overall) | 20 | 7 | 27 | 20.70 |
| **Post-Test** | Spatial Dimension | 20 | 5 | 9 | 8.15 |
| | Numerical-Spatial Dimension | 20 | 0 | 7 | 5.05 |
| | Mental Rotation | 20 | 2 | 7 | 4.80 |
| | Area Measurement | 20 | 0 | 5 | 4.10 |
| | Spatial Dimension | 20 | 8 | 27 | 22.10 |

When examining the pre-test results presented in Table 7, it was observed that participants obtained a minimum of 7 and a maximum of 27 points on the overall Spatial Visualization Test. The minimum-maximum score ranged for the sub-dimensions of the test was observed as 2–9 for the spatial dimension, 2–7 for the numerical-spatial dimension, 1–8 for the mental rotation dimension, and 0–5 for the area measurement dimension. When examining the post-test results of participants in the research group, it was determined that they scored between 8 and 27 points on the overall spatial visualization test. In the post-test application, the minimum-maximum score ranges for the sub-dimensions were 5–9 for the spatial dimension, 0–7 for the numerical-spatial dimension, 2–7 for the mental rotation dimension, and 0–5 for the area measurement dimension.

In the pre-test application, the participants' mean scores for the spatial, numerical-spatial, mental rotation, and area measurement sub-dimensions of the spatial visualization test were 7.30, 5.05, 5.05, and 3.30, respectively, while the overall mean of the test was calculated as 20.70. In the post-test application, these means were determined as 8.15 for the spatial dimension, 5.05 for the numerical-spatial dimension, 4.80 for the mental rotation dimension, and 4.10 for the area measurement dimension, with an overall test mean of 22.10.

It was observed that participants' post-test mean scores increased relative to pre-test application in the spatial and area measurement sub-dimensions, as well as in the overall spatial visualization test. However, although the group's pre-test and post-test means for the numerical-spatial dimension were identical, the post-test means for the mental rotation dimension were lower. The statistical significance of the differences between participants' pre-test and post-test means on the spatial visualization test was assessed using the Wilcoxon Signed Ranks Test, and the results are presented in Table 7.

**Table 7**

*Comparison of Spatial Visualization Test Pre-Test and Post-Test Means of Students Participating in the Activity*

| | **Spatial Dimension** | **Numerical-Spatial Dimension** | **Mental Rotation** | **Area Measurement** | **Spatial Visualization Test** |
|---|---|---|---|---|---|
| **Z** | -2.234 | -.183 | -.695 | -2.683 | -2.133 |
| **p** | .025 | .855 | .487 | .007 | .033 |

As indicated in Table 7, it was observed that the significance values for the spatial and area measurement sub-dimensions, as well as the overall spatial visualization test, obtained from the Wilcoxon Signed Ranks analysis applied to the pre-test and post-test scores, were .025, .007, and .033, respectively. Since these values were less than .05 ($p$<.05), the group's pre-test and post-test scores differ significantly in these two sub-dimensions and in the overall spatial visualization test. The significance values of the Wilcoxon Signed Ranks analysis applied to the pre-test and post-test scores for the numerical-spatial and mental rotation sub-dimensions of the research group's spatial visualization test were .855 and .487, respectively. Since these values were greater than .05, there was no statistically significant difference between the group's pre-test and post-test scores in the numerical-spatial and mental rotation sub-dimensions.

In summary, the six-week brain teaser activity carried out with the research group contributed, in general, to the development of the group's spatial visualization skills. In the mental rotation sub-dimension of the spatial visualization test, the group's mean in the post-test application was lower than the pre-test mean; however, this difference was not statistically significant. In the numerical-spatial sub-dimension, the group's pre-test and post-test means remained unchanged. In line with these results, it can be stated that the brain teaser activities performed did not positively contribute to the group's numerical-spatial and mental rotation skills.

## Discussion, Conclusion and Recommendations

In this study, a six-week, 12-hour activity program was designed using geometry-based brain teasers to investigate their effect on secondary school students' geometric thinking levels and spatial visualization skills. Upon examining the research findings, it was observed that the brain teaser activity resulted in a statistically significant increase in the students' geometric thinking levels. This finding is consistent with the results of the study by Dokumacı Sütçü (2018), which examined the effect of geometric-mechanical brain teasers on the geometric thinking levels of pre-service teachers. In that study, Dokumacı Sütçü (2018) conducted a nine-week activity program for two experimental groups, using both concrete materials and digital versions of the Q-Bitz Extreme, Architecto, and Katamino games. As a result of the research, there was a positive and statistically significant difference in the geometric thinking levels of both groups.

Furthermore, the games selected by Dokumacı Sütçü (2018) show similarities with the games included in this study. The Q-Bitz Extreme game is similar to the "Motif" game in this study, as it is a pattern-making game. The Architecto game is identical to the Equilibrio game, and the Katamino game is identical to the Penta Block game, albeit with different names. Consequently, it can be stated that games involving pattern formation (Q-Bitz Extreme, Motif), constructing structures with 3D geometric objects (Architecto-Equilibrio), and filling areas using geometric shapes of different sizes (Katamino-Penta Block) contribute positively to individuals' geometric thinking levels. Similarly, Karalı and Taşkesen (2022), in their study conducted with 3rd-grade primary school students using Tangram and Soma Cube games, concluded that these games increased students' 3D geometric thinking skills and academic achievement. In this context, the opportunities provided by the Tangram, Pentomino (Penta Block), and Geoboard games in the current research, which allowed students to create new constructions by bringing parts together and to discover the properties of shapes, can be identified as the source of this development.

The views of Van Hiele (1999), who states that games have an important place in geometry, support these findings. Van Hiele (1999) states that activities performed with pattern blocks enrich children's visual structure repository, enabling them to acquire knowledge about shapes and their properties. Additionally, placing pieces to fill an area in games provides experiences related to the area, while direct comparison allows students to discover relationships between pieces (Van Hiele, 1999). It can be stated that the brain teaser activities, specifically the Tangram and Geoboard games played in this study, had a positive impact on students' geometric thinking levels. Van Hiele (1999) also draws attention to the fact that games like Tangram should be readily available to teachers, stating that such games give teachers the chance to observe how students use parts and to reveal how they think about shapes.

Another finding of the research is that geometry-based brain teaser activities positively affected secondary school students' spatial visualization skills in general. This finding aligns with the results of many studies in the literature (Bakraç et al., 2023; Çilingir Altıner, 2018; Demirkaya & Masal, 2017; Dokumacı Sütçü & Oral, 2020; Yang & Chan, 2010; Zeybek & Saygı, 2019). Although the studies by Demirkaya and Masal (2017) and Dokumacı Sütçü and Oral (2020) are similar in that they examine the effect of geometric-mechanical brain teasers on the spatial skills of secondary school students, they differ in terms of the games selected for the activity phase. The Tangram game used by Demirkaya and Masal (2017) and the Architecto (Equilibrio) and Katamino (Penta Block) games used by Dokumacı Sütçü and Oral (2020) show similarities to the games used in this study. The findings of the current research are consistent with those of previous studies, which determined that engaging in brain-teasing activities had a positive effect on students' spatial skills.

Furthermore, games such as Penta Block and Motif used in the research are brain teasers based on part-whole relationships. This supports the positive relationship identified by Çilingir Altıner (2018) between puzzle games and spatial visualization. Similarly, the finding by Yang and Chan (2010) that the Pentomino game improves students' spatial abilities demonstrates the effectiveness of the Penta Block game used in this research in this regard. The findings of Bakraç et al. (2023), which revealed a significant disparity in the spatial orientation abilities of the experimental group within the context of their study employing the Fantastic Cubes game, are consistent with the results of the present study.

In the findings of this research, while a statistically significant positive difference was detected in students' overall spatial visualization skills as a result of the brain teaser activities, no significant difference was found in the numerical-spatial and mental rotation sub-dimensions of spatial visualization. These results are supported by the findings of Özcan et al. (2016), who investigated the correlation between university students' gaming habits and their spatial skills. While the findings of that research indicate no significant relationship between students' 3D digital game experiences and their mental rotation test scores, it was determined that students' spatial visualization test scores increased as their gaming experience increased.

In this regard, to improve students' mental rotation abilities, a study could be planned that extends over a longer period and utilizes more diverse games. This is because mental rotation is considered the most complex component of spatial visualization with the highest cognitive load (Sorby, 1999). For instance, Crissey's (2023) study observed that groups utilizing a combination of physical manipulatives and technology, such as GeoGebra and Sketchpad, engaged in activities spanning a duration of eight weeks. In this research, it was considered that the 6-week activity process, covering a total of 12 hours, may have been insufficient to improve students' mental rotation skills.

Teachers can use geometric-mechanical brain teasers in their lessons to increase students' spatial skills and geometric thinking levels. Furthermore, students can be assigned construction tasks on a geoboard styled as brain teaser questions. Various construction tasks that can be performed on a geoboard can be added to textbooks as activities. Since a geoboard can be printed on paper, it is more accessible and cost-effective compared to other brain teaser games. Therefore, various tasks that can be performed on a geoboard can be communicated to teachers and pre-service teachers, and their use in lessons can be encouraged.

In this research, a convenience sampling method was adopted, and the study group was determined as the students attending the "Mind and Intelligence Games Workshop" held at the secondary school where the primary researcher works. Due to the school's location in a rural area under the Ministry of National Education, as well as the limited number of students and physical constraints, it was practically impossible to establish an independent control group. Since there was no control group in this study, it is difficult to state that the significant increase observed in favor of the post-test in students' Van Hiele geometric thinking levels, and spatial visualization skills stems solely from the geometry-based brain teasers implemented. The development observed in students might have been influenced by external factors, such as natural cognitive maturation during the six-week process or pre-test familiarity resulting from test repetition. Therefore, although the findings obtained indicate a positive effect of brain teasers on spatial skills, this situation should be considered within the framework of limitations necessitated by the research design and sampling method.

In this context, for future research, the effect of brain teasers on Van Hiele geometric thinking levels and spatial visualization skills could be investigated through true experimental or quasi-experimental studies featuring experimental and control groups. Additionally, the utilization of larger sample sizes could offer more precise insights into the impact of the brain teasers used throughout the process. Conducting mixed-method research, in which quantitative tests are supported by clinical interviews and participant observations, would provide a significant contribution to the literature.

* This research was conducted with the approval of the Pamukkale University Social and Human Sciences Research and Publication Ethics Committee (dated 27.12.2024, number 68282350/2024/24) and the permission of the Denizli Provincial Directorate of National Education.

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