HOW AND WHY DOES THE MULTIPLICATION METHOD DEVELOPED BY THE RUSSIAN PEASANTS WORK?[1]

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ABSTRACT

In this article, an activity designed and implemented to improve both procedural knowledge and conceptual knowledge of multiplication is introduced. The students were physically and mentally active while exploring a multiplication method developed by the Russian peasants. They not only explained why and how the method works, but also extended the method. The activity was implemented at a Science and Art Center located in one of southwestern cities in Turkey. The activty was implemented in two classes having 11 and 15 students respectively. The lesson took an hour. The students used unit cubes and focused on the area meaning of multiplication to comprehend the new multiplication algorithm that they have learnt. The effectiveness of the lesson was assessed using observation notes of the teacher and worksheets completed by the students. The assessment results indicate that the lesson supported the conceptual and procedural understanding of the students.
Keywords: Russian peasant multiplication method, conceptual knowledge, procedural knowledge.

RUS KÖYLÜLERİN GELİŞTİRDİĞİ ÇARPMA YÖNTEMİ NASIL VE NEDEN ÇALIŞIYOR?

ÖZ

Bu makalede, çarpma konusunda öğrencilerin hem işlemsel bilgilerini hem de kavramsal bilgilerini geliştirmek amacıyla tasarlanan ve uygulanan bir etkinlik tanıtılmıştır. Öğrenciler, fiziksel ve zihinsel olarak aktif bir şekilde, Rus köylüleri tarafından geliştirilen bir çarpma yöntemini incelemişler, yöntemin neden ve nasıl çalıştığını açıklamışlar ve yöntemi genişletmişlerdir. Etkinlik güney batı illerimizden birisinde bulunan bir Bilim Sanat Merkezi'nde uygulanmıştır. Uygulama, 11 ve 15 kişiden oluşan iki ayrı sınıfta yapılmıştır. Uygulama süresi 1 saattir. Etkinlikte öğrenciler, birim küpleri kullanarak ve çarpmanın alan anlamı üzerine odaklanarak öğrendikleri yeni çarpma algoritmansını anlamlandırmışlardır. Dersin etkililiği, öğretmen tarafından tutulan gözlem notları ve öğrencilerin doldurduğu çalışma kağıtları kullanılarak değerlendirilmiştir. Dersin genel olarak öğrencilerin kavramsal ve işlemsel bilgisini geliştirdiği tespit edilmiştir.
Anahtar kelimeler: Rus köylü çarpma yöntemi, kavramsal bilgi, işlemsel bilgi.

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INTRODUCTION

Being successful in today's mathematics lessons means to have high-level thinking skills such as reasoning, generalizing, explaining the reasons behind the rules or formulas, and solving problems, beyond solving the drill and practice questions (National Council of Teachers of Mathematics [NCTM], 2014; Olkun & Toluk Uçar, 2009). For this reason, recent mathematics curricula include the mathematical processes such as reasoning, connecting, problem solving, and modeling in their introduction sections, and suggest that mathematics teaching should be based on these skills (Ministry of National Education [MoNE], 2005; MoNE, 2017; National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). This article describes a middle school-level activity in which students reason mathematically and express their ideas using multiple representations. In the activity, students examined a multiplication method developed by the Russian peasants, explained why and how the method works, and extended the method.

One of the goals of an effective mathematics lesson is that students acquire *procedural knowledge* and *conceptual knowledge* (Kilpatrick, Swafford, & Findell, 2001; Olkun & Toluk Uçar, 2009). *Procedural knowledge* allows application of formulas and/or rules fluently and responds to the question of how a rule works. On the other hand, *conceptual knowledge* allows to explain the concepts underlying these applied rules, and responds to the question of why a rule works. A student who has conceptual knowledge can relate mathematical concepts to each other. For example, if a student who knows how to calculate the area of a parallelogram, can make an argument about the area of a triangle such as “Each triangle is half of a parallelogram, because if you turn over the copy of any triangle and add the copy of the triangle to itself, you get a parallelogram. Therefore, the area of the triangle is half of the area of the parallelogram.” it can be said that this student has a conceptual knowledge of the area of the triangle. If the student can apply the triangle area formula and can correctly calculate the area of a given triangle, then it can be said that the student has procedural knowledge of triangle area. Both kinds of knowledge are important, and students should be expected to have both knowledge types. However, in some mathematics courses, procedural knowledge is more emphasized and the mathematical rules are memorized without being understood by the students (Birgin & Gürbüz, 2009). In such cases, after a while, students confuse the rules and perform meaningless procedures. Even worse, students may begin to perceive mathematics as a discipline composed of meaningless rules.

This activity aimed to develop both procedural knowledge and conceptual knowledge of the students. The activity was designed and implemented in which the students were physically and mentally active to participate in the lesson. This paper describes the implementation of the activity and assessment of students’ learning. The activity aims to develop students’ conceptual and procedural knowledge about a multiplication method developed by the Russian peasants. The activity focuses on the area model of multiplication (Lee, 2014).

A targeted sub-goal in this activity is to raise students’ awareness of the fact that mathematics was developed and is still developing with the contributions of different cultures. For example, if the multiplication, the topic of the activity, is taken into consideration, it is known that the multiplication process was performed in Egypt and Russia by doubling method, in India by the perpendicular and crossmatch method, and in Japan by using abacus (Lin, 2007). This activity aims to help students recognize that mathematics emerges as a natural need in all cultures and realize that different mathematical methods were developed in different cultures. Finally, another sub-goal of the activity is to discuss with the students that one reason for the development of mathematics is the daily life needs of people. For instance, the multiplication method examined in the activity was developed to meet the daily needs of the villagers. As part of the activity, it is aimed to highlight the aspect of mathematics that produces solutions to the needs of people in everyday life by discussing how the Russian villagers developed the multiplication method in order to easily count their agricultural products.

ACTIVITY IMPLEMENTATION

Contextual Information

The activity was implemented at a Science and Art Center located in one of the southwestern cities in Turkey. The permission needed for implementation was obtained. The Science and Art Center has weekday and weekend groups. This activity was first applied with 15 students attending the weekend sessions at the center. Of these students; one is in the fifth grade, eight are in the sixth grade and six are in the seventh grade. The activity was then applied with a class of 11 students, consisting of fifth and sixth graders, who attended the center on weekdays.

The activity was similarly applied in both groups. The minor differences in practice are indicated in the next chapter "Teaching-Learning Process." In both applications, activity worksheets completed by the students were collected and used for data analysis. During the first application, the teacher took notes for important points to use for subsequent reflective evaluation. For this reason, the next chapter is mostly based on the first application.

The lesson lasted approximately 1 hour in both applications. The standards of the Science and Art Center addressed by the activity are:
1) Understand how to multiply two whole numbers using the Russian Peasant Method.
2) Multiply two whole numbers using the Russian Peasant Method.

The relevant standards of the Ministry of National Education mathematics curriculum (MoNE, 2017) addressed by the activity are as follows:
M.5.1.2.4. Multiply two whole numbers with up to three digits.
M.5.1.2.12. Solve problems involving four operations.
M.6.1.1.4. Solve and construct problems that require four operations with whole numbers.

Although the activity was mostly designed towards the fifth or sixth grade students, it is not limited to a specific grade level, as it involves a problem solving process. Teachers can use or adapt the activity in different classes according to the readiness of their students.

It is quite easy to apply the activity in terms of material. The required materials are unit cubes, pencil, and the activity worsheet. The worksheet is presented in Appendix 1.

Teaching-Learning Process

In the introduction to the lesson, the teacher explained to the students that the Russian villagers had developed a multiplication method different from the method that we commonly use today, and in this method the overall approach is to divide the first multiplier continuously into 2 and multiply the second multiplier by 2. In order to assess the level of procedural knowledge at the beginning of the lesson, the students were asked whether they had heard of this method before and whether they knew how to use it. Three students said that they had heard about such a method but they did not remember exactly the procedures used in the method. Therefore, it can be argued that all students started the lesson without the procedural and conceptual knowledge of the topic.

Another question posed to the students at the introduction part was the question why the Russian villagers might have needed to develop a multiplication method. There were different answers. Some students responded "I do not know" and some said "To count their products." When asked to elaborate the latter type of answers, a student gave an example of counting agricultural products as "They might put the potatoes into sacks and need to multiply to find the number of total potatoes." The teacher added that mathematics is sometimes developed in this way, people created mathematics to meet the needs of everyday life. A goal in this part of the lesson was to raise awareness in general about the nature of mathematics.

In the second phase of the lesson, firstly, each student was given the activity worksheet given in Appendix 1. The first question was solved and explained by the teacher on the board, and the students performed the same procedures on their worksheets. The multiplication algorithm developed by the Russian peasants works as follows. The multiplied numbers are written side by side as in Table 1. The first multiplier is continuously divided into 2, while the second multiplier is doubled continuously. If the number is odd in the division process, it is reduced by 1 and divided into 2. When the division by 2 ends with 1, division and multiplication processes stop. The product is found by adding the numbers in the second column corresponding to the odd numbers in the first column.

Table 1. Multiplication Method Used by Russian Peasants

12	23
6	46
3	92
1	184

In Table 1, the odd numbers in the left column are 3 and 1, and in the right column the corresponding numbers are 92 and 184. During the lesson, for the even numbers in the first column, the students put a line on the row including the even number to exclude them from the algorithm. The result of the multiplication is 276, found by adding 92 + 184. The most prominent advantage of this method is using the addition operation and multiplication and division by 2. Students usually multiply or divide by 2 easier than other multiplications and divisions. A disadvantage of the method is that the process can be very long and become more complicated than the conventional multiplication algorithm based on the multiplied numbers.

Students completed the second question on the worksheet. The goal here was to help them develop procedural knowledge. As will be detailed in the next section "Measurement and Evaluation" students in general correctly used this new method of multiplication. An example of a student's response is given in Figure 1.

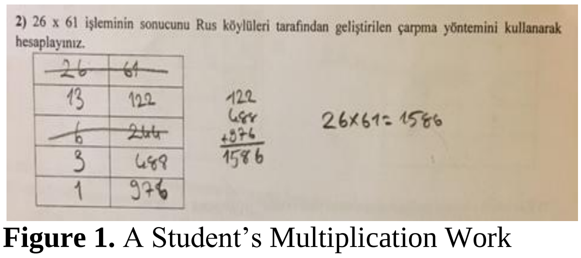

Figure 1. A Student's Multiplication Work

The third question on the worksheet (to explain the method to a friend who was absent) was asked to enable the students to represent the new method they have learned verbally. Writing is an action that helps students organize the knowledge in their minds (NCTM, 2000). Another goal was to measure the students' ability to explain the procedural knowledge with words. While some students gave short answers such as "do the operations," some of them preferred to make explanations by multiplying numbers, but most students defined the steps of the algorithm. Such a student response is given in Figure 2. The answer of this learner is:

> The first multiplier is divided into 2 until getting 1. If it is not divided by 2, reduce it by 1 and then divide into 2. The second multiplier is multiplied by 2. Even numbers are crossed out and the numbers on the right are added.

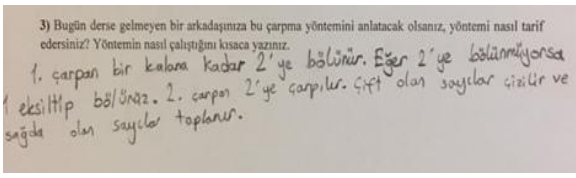

Figure 2. A Student's Explanation of the Multiplication Method

Some volunteer students read their responses for the third question to whole class and students made comments on others' explanations. The teacher stated that these instructions are named algorithm and in this lesson they learned a new multiplication algorithm.

The fourth question of the worksheet (Why does this method give the right answer?) was asked to assess the conceptual knowledge of students about the multiplication method. In fact, it was expected that the students would not be able to answer this question because no inquiry was done so far. As expected many students responded with expressions such as "How can I know this?" But some students stated that one side was multiplied by 2 and the other side was divided into 2, creating a balance. Nevertheless, as one of these students stated "But I do not know how it works when 1 is subtracted" there was not any student who could fully explain the algorithm.

The previous part of the lesson focused on improving the procedural knowledge about the multiplication method used by the Russian villagers. The next part aims to develop conceptual knowledge about this multiplication method. Students formed groups of three and each group received 60 unit cubes. The fifth question aimed at recalling the area model of multiplication and was successfully answered. In the first application of the activity, for simplicity, the students were asked to represent 2x4 by using the unit cubes. In the second application, the transition to the sixth question was facilitated by directly asking for a representation of 8x5. In order to use a common language during class discussion, the students were told to represent the height of the rectangle by the first multiplier.

In the sixth question, the students were asked to perform the multiplication both using the table and representing each row using the unit cubes to make sense of the multiplication method. While the groups were working with unit cubes, the teacher walked among the groups and helped the students as needed. The most challenging part was to represent the meaning of multiplication with unit cubes as the algorithm was applied. For example, while the students formed the correct rectangle for 8x5, the students had difficulty in forming the rectangle for 4x10 in the second row. The teacher supported students' reasoning by asking questions such as "What is the meaning of 8x5?" "What is the meaning of 4x10?" "How can you transform the first rectangle so that you represent 4x10?" When more than half of the class completed the first table in the sixth question, the teacher drew unit cubes on the board and connected the tabular and pictorial representations. The students also provided explanations. The groups were asked to complete the other three tables in the sixth question in a similar way. The unit cubes formed by a group for 6x5 are shown in Figure 3, Figure 4, and Figure 5, respectively.

The multiplication table that correspond to shapes shown in Figures 3-5 is given in Table 2. In Figure 3, there is a rectangle formed to represent 6x5. The purpose of this multiplication is to find out 6 groups of 5, so the rectangle shows 6 groups of 5 unit cubes. Figure 3 corresponds to the first row of Table 2.

Figure 3. Representation of 6x5 with Unit Cubes

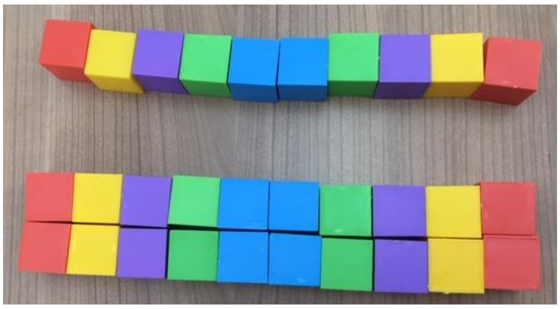

Figure 4. Representation of 3x10 with Unit Cubes

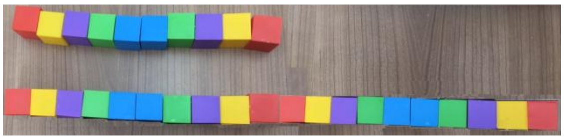

Figure 5. Representation of 1x10+1x20 with Unit Cubes

Table 2. Calculating 6x5 with the Russian Peasants' Multiplication Method

6	5
3	10
1	20

Figure 4 represents the second row of Table 2. The first rectangle was divided into two parts and the parts were connected to each other in the longitudinal direction to form a rectangle representing 3x10. Since this rectangle can not be divided into 2 again, one row is separated from itself. In fact, one group of 10 unit cubes are separated. This row should not be forgotten when finding the total number of cubes. Figure 5 shows a 1x20 rectangle formed by horizontally dividing the rectangle representing 2x10, and a 1x10 rectangle that was separated in the previous step. The 1x20 rectangle in Figure 5 corresponds to the last row of Table 2. To find the result of the multiplication, that is to find the total number of unit cubes, the number of unit cubes constituting the rectangles in Figure 5 are summed (10 + 20). These numbers correspond to odd numbers in the first column of Table 2. The numbers that correspond to even numbers in the first column are not added because the rectangles formed by these numbers are divisible by 2 and do not exclude any unit cube.

The seventh question on the worksheet asks students to explain why this method works. Even though all groups successfully completed the previous question using unit cubes, there were students who were reluctant to write in this question. They stated that they could speak instead of writing. For this reason some students have left this question blank. In the second implementation of the activity, the teacher was more assertive and all students wrote their thoughts. The teacher selected two student responses that explained the method comprehensively to be read and discussed in class. The teacher supported students' ideas by drawing appropriate visuals on the board. Figure 6 shows one of the student responses shared in class. The student has written the following statements:

> The value of the number does not change when you multiply one and divide the other. The numbers that are reduced by one are odd and since we add them again, the result of the multiplication does not change. There is nothing that is removed or reduced.

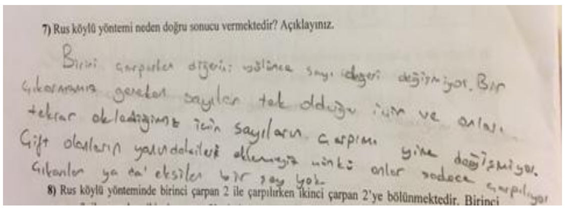

Figure 6. A Student's Answer to the Seventh Question

It was the end of the lesson when the seventh question was answered. At the closing of the lesson, the teacher expressed that different cultures have different contributions to mathematics and other multiplication algorithms are also available. For example, Japanese abacus is available on the internet and suggested that students investigate these different algorithms.

An extra question was written on the worksheet for groups completing the questions earlier. There were no such groups and the extra question was not answered in class. However, two students who were interested in this multiplication method worked on the additional question at a separate time and examined how the method worked in the case of multiplying and dividing by 3. Table 3 shows this method for 24x11. This time, the algorithm stops at 1 or 2 because these numbers are not divisible by 3. This algorithm can be done with unit cubes, but it will be explained with drawings to use a different representation. A 24x11 rectangle is drawn (Figure 7, Step 1). This rectangle is divided into three equal parts in accordance with the second row of Table 3, and the parts are added together to form a 8x33 rectangle (Figure 7, Step 2). To divide the rectangle horizontally into three equal parts, a rectangle with dimensions 2x33 is separated and from the remaining part, Step 3 in Figure 7 is obtained. The result of the multiplication, i.e., the total area, is obtained by calculating 2x33 + 2x99.

Table 3. Calculating 24x11

24	11
8	33
2	99

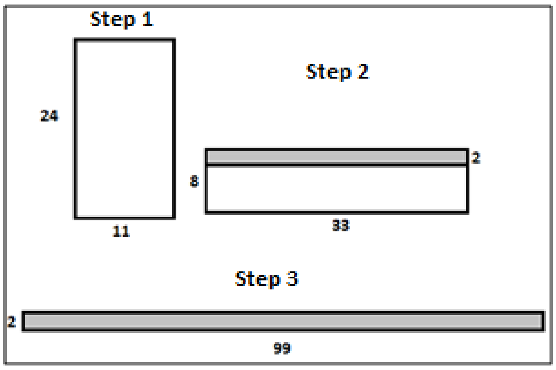

Figure 7. Pictorial Representation of the Calculation 24x11

Measurement and Evaluation

In this activity, it was aimed to develop students' conceptual and procedural knowledge of the multiplication method developed by the Russian peasants. Two assessment methods were used to determine if this goal was achieved or not. The first is that the teacher observed student learning during the lesson. According to the teacher's observation, all students successfully applied the method and all groups were able to represent the steps of the algorithm using unit cubes. Therefore it can be inferred that students constructed conceptual and procedural knowledge by participating in this activity.

The second method used for assessment purpose was to evaluate students' worksheets. This evaluation was performed from two perspectives: procedural knowledge and conceptual knowledge.

Procedural Knowledge. Since there were no students who knew this multiplication method at the beginning of the lesson, it can be said that the students started the lesson without procedural knowledge about this method. The first, second, and sixth questions on the worksheet give us information about the students' procedural knowledge as they require application of the algorithm. However, since as part of the sixth question the students used unit cubes and focused on making sense of the algorithm, some students did not write all of the steps on paper. In addition, since this question was answered as a group, there were groups where only one person's worksheet was used. For this reason, the first and second questions were used to assess students' procedural knowledge. The third question on the worksheet measures students' procedural knowledge in writing. Students' procedural knowledge and their ability of expressing procedural knowledge in writing were assessed using two separate four-point rubrics adapted from Van de Walle, Karp, and Bay-Williams (2013) and given in Appendix 2. The findings obtained using these 4-point rubrics are given in Table 4 and Table 5. Since students in both implementations performed similarly, the findings are presented for all students.

Table 4 shows that the students received almost full scores on implementing the algorithm. This finding suggests that the students developed procedural knowledge about the multiplication method.

Table 4. Procedural Knowledge Levels of Students

Mean Score of the Procedural Knowledge Scores	3.9

Table 5. Levels of Expressing Procedural Knowledge

Score	Frequency (Percent)
4	5 (19%)
3	15 (58%)
2	4 (15%)
1	2 (8%)

Since the students' level of expressing the procedural knowledge skills varied, the number of students in each score is given separately in Table 5. The mean score of expressing procedural knowledge is about 3 points (2.9). The students are overall successful in expressing procedural knowledge. However, the students are more successful in applying the algorithm compared to expressing the algorithm in writing.

Conceptual Knowledge. The fourth and seventh questions on the worksheet were asked to assess the conceptual knowledge of students. The fourth question was asked before the unit cubes were used, and the seventh question was asked after the students worked with unit cubes in order to understand the effect of working with unit cubes. The answers given by the students in both questions were evaluated with the rubric given in Appendix 3. Findings are presented in Table 6. It should be noted that both questions measure the ability of students to express their conceptual knowledge in writing. The questions require students to explain their thoughts in writing.

The scores of the fourth question in Table 6 (1 and 2 points) indicate that students did not develop conceptual knowledge sufficiently by only applying the algorithm. Teacher observations confirm this inference because for the fourth question many students stated that they would not know why the algorithm worked. Some students left this question blank.

Table 6. Levels of Expressing Conceptual Knowledge

Score	Frequency (Percent) – Fourth Question	Frequency (Percent) – Seventh Question
4	0	4 (15%)
3	0	3 (12%)
2	10 (38%)	13 (50%)
1	7 (27%)	0

The scores for the seventh question in Table 6 (2, 3, and 4 points) show that, although not at the desired and expected level, there were students who could explain why the algorithm worked after having worked with unit cubes. Aligned with this finding, the teacher observed that all groups successfully represented how the algorithm worked with unit cubes. One reason for not having the scores as high as expected might be students' difficulty in expressing their thoughts in writing. Another reason might be that after the discussion of the seventh question, the students submitted their papers because the lesson ended. It is likely that student learning continued after they wrote the answers to the seventh question, because during the class discussion the students shared their ideas with the whole class and the teacher supported these ideas by drawing the related visuals on the board. To sum up, both the scores in Table 6 and the teacher's observations show that the activity contributed to the conceptual knowledge of the students.

CONCLUSIONS and SUGGESTIONS

In this article, an activity is shared where students were mentally and physically active throughout the lesson. The students expressed their mathematical thoughts by speaking, writing, and using materials. They explored a multiplication algorithm developed by the Russian peasants and explained why and how this algorithm worked. Some interested students extended the algorithm.

The assessment results indicate that this activity improved the conceptual and procedural knowledge of the students about the multiplication method developed by the Russian peasants. Lesson observation notes and evaluation results show that students were reluctant to write in mathematics lessons and were not very successful in explaining their opinions in writing. The students were more successful in applying the algorithm than by writing about the algorithm.

Similarly, students did not perform as well in writing as they could successfully demonstrate why the algorithm worked with unit cubes. In fact, similar findings were reported in the literature (Atasoy & Atasoy, 2006; Seo, 2015). For example, Atasoy and Atasoy (2006) found that students had difficulty at first in journal writing activities in mathematics class, and their independent writing skills improved over time. Some teachers believe that it is not necessary to write in a mathematics class, and students who first encounter writing activities may be reluctant to write (Seo, 2015). While writing and doing mathematics may seem like unrelated activities (Burns, 2004), in fact writing activities in mathematics lessons can contribute to the procedural and conceptual knowledge of students (NCTM, 2000; Yılmaz, 2015). For this reason, students should write regularly in mathematics lessons. Initially, writing activities could be done as a group, particularly for students having difficulty in writing. Another suggestion is to give students the opportunity to rewrite after class discussion. In this way, students can develop their writing by blending the ideas of other students with their own understanding.

Another suggestion for future applications of this activity is using the activity with the eighth grade students. By using concepts such as algebraic expressions and factoring, eighth grade students can interpret the Russian peasant multiplication method at a level different from the current practice.

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Appendix 1

Worksheet

THE MULTIPLICATION METHOD DEVELOPED BY THE RUSSIAN PEASANTS

In this lesson, you will investigate a multiplication method developed the Russian peasants.

1) Follow the teacher's instructions to calculate 12 x 23 by completing the following table.

12	23

2) Calculate 26 x 61 by using the multiplication method developed the Russian peasants.

3) How would you describe the method to one of your friends who did not come to class today? Briefly describe how the method works.

4) Why do you think this method gives the right answer?

5) Show 8x5 with unit cubes. Focus on the meaning of the operation. Hint: You should build a rectangle.

6) Calculate the following multiplications by the method of Russian villagers. Show the operation with unit cubes for each row. Try to understand why the method works.

8x5	
8	5

6x5	
6	5

7x4	
7	4

5x13	
5	13

7) Why does the Russian peasants' multiplication method work? Please explain.

Extra Question: In the Russian peasants' multiplication method, the first multiplier is divided into 2, while the second multiplier is multiplied by 2. If we divide the first multiplier by 3 and multiply the second multiplier by 3, would we find the right answer? Examine this case with various examples. If you reach a rule, write down your rule and be ready to explain your findings to the whole class.

Appendix 2

Rubrics Related to Procedurel Knowledge

The Rubric Used to Assess Procedural Knowledge

Successful		Not Yet	
4 Points: Excellent	3 Points: Proficient	2 Points: Marginal	1 Point: Unsatisfactory
The algorithm is applied correctly in all questions and the result of the multiplication is correctly calculated.	The algorithm was applied correctly but the result is wrong because of a simple calculation error.	There are no completed procedures to determine that the algorithm is fully learned. More teaching is required.	There is an attempt to apply the algorithm but little or no success.

The Rubric Used to Assess Expressing Procedural Knowledge

Successful		Not Yet	
4 Points: Excellent	3 Points: Proficient	2 Points: Marginal	1 Point: Unsatisfactory
The following points related to the algorithm are mentioned: 1) The first multiplier is repeatedly divided by 2, the second multiplier is multiplied by 2. 2) If the first multiplier is odd, it is reduced by 1 and then divided into 2. 3) When the first multiplier is 1, the algorithm stops. 4) The result of the multiplication is found by adding the numbers in the second column that correspond to the odd numbers in the first column.	Two or three of the four elements required to score four points are indicated. Explanations made through samples are in this category.	The first of the four elements required to score four points is indicated.	Instead of the main principles of algorithm, superficial elements are written. For example, “Construct a table.”

Appendix 3

The Rubric Used to Assess Conceptual Knowledge

Successful		Not Yet	
4 Points: Excellent	3 Points: Proficient	2 Points: Marginal	1 Point: Unsatisfactory
The reason why the algorithm works is explained considering both cases when the first multiplier is odd or even. Descriptions may include visual or verbal representations.	The reason why the algorithm works is explained by considering case when the first factor is even. Descriptions may include visual or verbal representations.	The reason why the algorithm works is explained in general, without specifying the cause, such as multiplying and dividing balance each other.	A meaningful reason why the algorithm works is not suggested.