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Integrating Inquiry-Based Learning and Hypothetical Learning Trajectories: An Activity-Based Study

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This study aimed to enable fifth-grade students to meaningfully discover and learn concepts related to polygons and the relationships among these concepts through activities based on the integration of Inquiry-Based Learning (IBL) and the Hypothetical Learning Trajectory (HLT) and to improve their conceptual understanding of polygons. A total of 27 students (14 girls and 13 boys) participated in this research, which was conducted using a design-based approach. The data collection instrument prepared to evaluate the research process was administered as a pre-test and post-test. At the end of the study, it was determined that the activities prepared in accordance with the integration of IBL and HLT positively improved the participants' conceptual understanding of polygons. Furthermore, Furthermore, suggestions were presented regarding the positive aspects of the process and areas requiring improvement based on the implementation of activities integrating IBL and HLT.

Keywords: Inquiry-based learning, hypothetical learning trajectory, polygons.

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Introduction

Polygons are one of the fundamental concepts of geometry. A polygon is defined as a shape formed by line segments that connect three or more non-collinear points in a plane pairwise. For a shape to be defined as a polygon, it must satisfy the properties of "the presence of at least three non-collinear points in a plane and the shape being closed." In teaching the concept of polygons, in addition to emphasizing the properties of polygons through visuals, it is necessary to establish relationships based on the definition of the concept (Güven et al., 2019). In the 2024 middle school mathematics curriculum in Turkey, the concept of polygons begins to be introduced starting from the fifth-grade level. At the fifth-grade level, the theme "Geometric Shapes" includes two learning outcomes: "Interpreting polygons as closed shapes formed by consecutively intersecting lines in a plane" and "Reflecting experiences related to the properties of polygons." The aim of these learning outcomes is for students to comprehend the basic properties of polygons and to interpret polygons (Ministry of National Education [MoNE], 2024a). In general, at the middle school level, the focus is on convex polygons, and special polygons are studied. In addition, it is important for students to understand the existence of non-convex polygons, particularly when determining whether a circle is a polygon (Argün et al., 2020).

Inquiry-Based Learning

Inquiry-Based Learning (IBL) emerged from theoretical developments associated with the work of Piaget and Vygotsky, who emphasized the importance of constructivist approaches. The main aim of IBL is to move students from a passive recipient position to one in which knowledge is discovered and constructed through direct experience, social interaction, critical thinking, and research. The use of IBL in mathematics education aims to help students perceive mathematics not as a set of fixed procedures but as a process of discovery, to develop a deep understanding of mathematics, and to establish an authentic connection with the subject (Lazonder, 2013; Vasuki et al., 2016).

IBL consists of inquiry phases that form the inquiry cycle (Pedaste et al., 2015). In their study, Pedaste et al. (2015) proposed a model consisting of five general phases and nine sub-phases for IBL. These phases are orientation, conceptualization, investigation, conclusion, and discussion. The researchers stated that the IBL process began with orientation, progressed from conceptualization to investigation, and ended with conclusion and discussion. Orientation refers to the process of stimulating curiosity about a topic and addressing a learning difficulty through a problem statement. The conceptualization phase involves the formulation of theoretical questions and hypotheses. Therefore, this phase is divided into two subcategories: questioning and hypothesis generation. The investigation phase includes processes such as exploration, experimental design, data collection, and analysis. Accordingly, this phase includes three subcategories: exploration, experimentation, and data interpretation. The conclusion phase involves drawing conclusions from data, making inferences based on the data, and comparing these with the research questions. The discussion phase includes presenting, reflecting on, and evaluating findings from specific stages of the research process or from the entire inquiry cycle

in relation to other findings. Therefore, this phase consists of two subcategories: communication and reflection (Pedaste et al., 2015).

In IBL, the teacher assumes a supportive and facilitative role to encourage students to ask questions, explore the subject, and construct their own understanding. This approach makes the learning process more dynamic and student-centered, with the teacher providing guidance and support when necessary. In mathematics education, IBL aims to help students learn mathematical concepts, formulas, or algorithms not by memorization but by exploring problems, forming hypotheses, and reflecting on their own reasoning (Vasuki et al., 2016).

Learning Trajectories-Hypothetical Learning Trajectory

Learning trajectories integrate theoretical principles and empirical evidence related to human knowledge and educational development and reflect the gradual and sequential structure of the learning process. Instructional practices aligned with developmental trajectories produce more effective and efficient pedagogical outcomes, and it is also accepted that adhering to a scientifically grounded instructional sequence has positive effects on student learning outcomes (Clements & Sarama, 2009; Jia & Zhu, 2025).

The main purpose of learning trajectories is to show the sequence in which a student acquires knowledge. The general assumption is that a student begins with the simplest level of knowledge and gradually progresses toward more complex knowledge. Learning trajectories encompass the progression from having no knowledge of a concept's properties to understanding all of its properties. Learning trajectories aim to maximize learning efficiency by aligning instruction or learning with the cognitive rules of students (Zhu, 2023). Learning trajectories provide teachers with opportunities to better understand students' levels of knowledge and thinking (Clements & Sarama, 2009).

Hypothetical Learning Trajectory (HLT) is an extended version of the concept of learning trajectories. It is defined as a concept that covers the holistic structure of instructional goals and the teaching techniques aimed at achieving these goals (Baroody et al., 2022). It is also described as a predictable sequence of structures that shows how knowledge progresses from an initial level to more complex levels (Confrey & Maloney, 2010). Simon (1995) used HLT to describe the process by which a student progresses from their initial point toward a specified learning goal. The use of the term "hypothetical" stems from the uncertainty inherent in each student's learning process (Sztajn et al., 2012). The term "hypothetical" is also used to express the teacher's prediction regarding the path that learning will follow. Although actual learning trajectories cannot be known with certainty in advance, they reflect an expected tendency. While students' learning processes are unique, they generally follow similar developmental paths (Simon, 1995).

In mathematics education, the general view is that HLT is an important tool in the development of mathematics instruction. According to this approach, instruction should not focus directly on the targeted higher-level knowledge but rather on developing the lower-level knowledge and skills that form the foundation for achieving it. Furthermore, instructional practices that take

the developmental sequence into account are considered more efficient and effective than project-based approaches that do not consider this sequence (Baroody et al., 2022).

HLT consists of three main components:

1. A specific mathematical goal that defines the direction of learning,

2. The developmental path that students follow to reach this goal, and

3. A set of instructional activities that help students progress along this path, along with predictions about how students' thinking and understanding will develop within the context of these activities (Clements & Sarama, 2009; Simon, 1995).

In the literature, there are a limited number of studies on the implementation of HLT in mathematics education. Among these studies, the work of Eroğlu and Tanışlı (2020) showed that HLT improved teachers' abilities to use representations and supported their ability to reflect these skills in instructional environments. In another study by Eroğlu et al. (2019), it was emphasized that the HLT used in the instructional process helped students learn topics conceptually. Wiryanto et al. (2024) found that students in the class where the HLT model was implemented learned mathematical concepts better and made significant progress in their learning processes. In addition, teachers participating in the study stated that lesson plans prepared according to HLT facilitated instruction. The results of these studies suggest that instruction supported by HLT is effective in enhancing students' conceptual learning. Rizki and Amir (2025) examined the effects of integrating IBL and HLT from a different perspective. They investigated the extent to which the integration of these two approaches contributed to students' understanding of the concept of volume. In the study, the fundamental structure of HLT was taken into consideration when designing the learning activities. Accordingly, the learning process comprised five stages: orientation, conceptualization, investigation, conclusion, and discussion. Each stage was designed to actively engage students in processes such as exploring phenomena, generating hypotheses, interpreting data, drawing evidence-based conclusions, and discussing their findings. At the end of the study, the researchers determined that activities integrating IBL and HLT made significant contributions to the development of students' skills in questioning, hypothesizing, interpreting, and spatial thinking.

Purpose of the Activity

The integration of IBL and HLT aims to develop students' conceptual understanding in a gradual and meaningful manner. While IBL promotes active student engagement through questioning, exploration, experimentation, and reflection within the instructional process, HLT provides a structured learning framework. The integration of IBL and HLT provides a structured framework that facilitates the gradual and meaningful development of students' conceptual understanding (Rizki & Amir, 2025).

This study aimed to enable students to meaningfully discover and learn concepts related to polygons and the relationships among these concepts through activities developed based on the integration of IBL and HLT and to improve their conceptual understanding of polygons. To

support this purpose, activities prepared through the integration of IBL and HLT were used in this study (Appendix-3).

Preparation of the Activity

The common text of the Maarif Model published in 2024 emphasizes that teaching and learning practices should include activities that ensure active student participation and support inquiry-based, deep, and meaningful learning, as well as instructional activities that support inquiry-based learning (MoNE, 2024b).

For the topic of polygons within the "geometric shapes" theme at the fifth-grade level, the curriculum includes two learning outcomes and six process components. Within the curriculum, the learning outcomes are "MAT.5.3.5. Interpreting polygons as closed shapes formed by consecutively intersecting lines in a plane" and "MAT.5.3.6. Reflecting experiences related to the properties of polygons." When these learning outcomes and their associated process components are examined, students are expected to recognize polygons, understand their basic properties, construct polygons, and make inferences about their side and angle properties. In this context, activities incorporating the integration of IBL and HLT were prepared for the study. Considering the fundamental components of IBL and HLT, an instructional design for teaching polygons was prepared in line with expert opinions (Appendix-1). When designing the activities, the stages of IBL, i.e., orientation, conceptualization, investigation, conclusion, and discussion, were first considered. For each stage, the learning outcome was identified to determine the learning objective of HLT, and the achieving the objective stage was determined to identify what was necessary to achieve this objective. Finally, the learning intervention stage was developed to provide a series of instructional activities that would help students progress along this pathway and to make predictions about how students' thinking and understanding would develop within the context of the learning activities.

Method

The research utilized a design research approach. However, it was not conducted within a full-fledged framework involving iterative redesign and re-implementation cycles of design research. Within the scope of the study, learning activities were designed and implemented based on the theoretical framework and principles of IBL and HLT, and the data obtained during the implementation process were analyzed. Consequently, the study's design process focused on the design and implementation phases. This design-based research followed the three-phase process identified by Akker et al. (2006). The first was the preparation phase of the study, in which a literature-supported HLT was designed and students' prior knowledge was analyzed. The second was the design phase, in which observations were conducted by implementing activities integrating IBL and HLT in the classroom. In the final phase, retrospective analysis, the findings obtained were compared with the assumptions at the beginning of the study. The main purpose of this design was to generate findings that could contribute to the development and improvement of learning processes (Akker et al., 2006).

Participants

The participants in the study were 27 middle school students (14 girls and 13 boys) attending fifth grade at a public school located in the Central Anatolia Region of Turkey. No prerequisite was required in the selection of the class. The class can be described as having a low socioeconomic and educational background. The study was implemented in a class deemed appropriate by the school administration. Therefore, the study used a convenience sample.

Data Collection Instrument

In the study, a data collection instrument consisting of five open-ended questions was used (Appendix-2). The first question of the data collection instrument was prepared to assess students' ability to recognize polygons and to determine whether they recognized the basic properties of polygons and how they expressed them mathematically. The second question was adapted from the study by Bernabeu et al. (2021). This question aimed to measure students' ability to apply and transform the concepts they had learned about polygons. In this question, students were expected to transform given shapes that lacked critical properties of polygons (e.g., a non-closed shape and a shape with a curved/non-linear side) into polygonal shapes. The third question was prepared to assess students' ability to classify polygons and recognize their properties. In this question, students were expected to use a protractor and a ruler to identify regular and irregular polygons and recognize their properties (Clements & Battista, 1992). The fourth question was prepared to assess students' ability to apply the concepts they had learned about polygons. The final question was prepared to identify possible misconceptions students might have regarding polygons. The data collection instrument and activities were prepared based on the relevant literature (Güven et al., 2019; MoNE, 2024c; Öztürk, 2020; Ulusoy, 2022; Zehir & Zehir, 2022). Two mathematics educators collaboratively developed the data collection instruments. Prior to the main study, a pilot implementation was conducted in one classroom, and the necessary revisions were made to the questions based on the pilot findings.

Data Analysis

The prepared measurement instrument was administered to students as a pre-test and post-test. To evaluate the responses to the instrument, students' answers were classified under three categories: correct (2 points), partially correct (1 point), and incorrect (0 points). The responses to the data collection instrument were first coded independently by two researchers. Then, the two researchers resolved any disagreements in coding through discussion. Therefore, each student received a total score ranging from 0 to 10 points for both the pre-test and post-test.

Ethics Committee Approval

This study complied with the "Directive on Scientific Research and Publication Ethics of Higher Education Institutions." Ethics committee approval for the study was obtained from Yozgat Bozok University (date: 9.10.2025, decision number:29/20).

Implementation of the Activity

The activity was implemented over seven class hours. In the first-class hour, the pre-test was administered to examine students' prior knowledge about polygons. The orientation activity

was implemented in the second-class hour, the conceptualization activity in the third, the investigation activity in the fourth, the conclusion activity in the fifth, and the discussion and reflection activity in the sixth. Finally, the post-test was administered in the seventh-class hour.

1. Orientation Activity

The orientation activity, corresponding to the stage of encountering a problem in IBL, was implemented as the first activity. The HLT learning goal of the orientation activity was to develop students' intuitive awareness of polygons and enable them to recognize the distinguishing properties of polygons. Visuals consisting of a triangle, square, pentagon, circle, open broken line (open polygonal chain), and self-intersecting shapes were distributed. The students were then asked to respond to the question, "Which ones are polygons? Why?" first individually and then through group discussion. In this process, the students were expected to classify shapes based on visual similarities, pay attention to side properties, and justify the condition of being closed. Within the HLT framework, the students were guided along reasoning paths based on "visual similarity/number of sides/being closed/having linear sides."

To achieve the HLT goal, a discussion environment was created, and the students explained why certain shapes are polygons and why others are not. During implementation, the majority of students initially classified shapes according to the number of sides. However, during discussion involving examples such as the "circle" and "open broken line," they began to identify criteria such as being closed and having line segments.

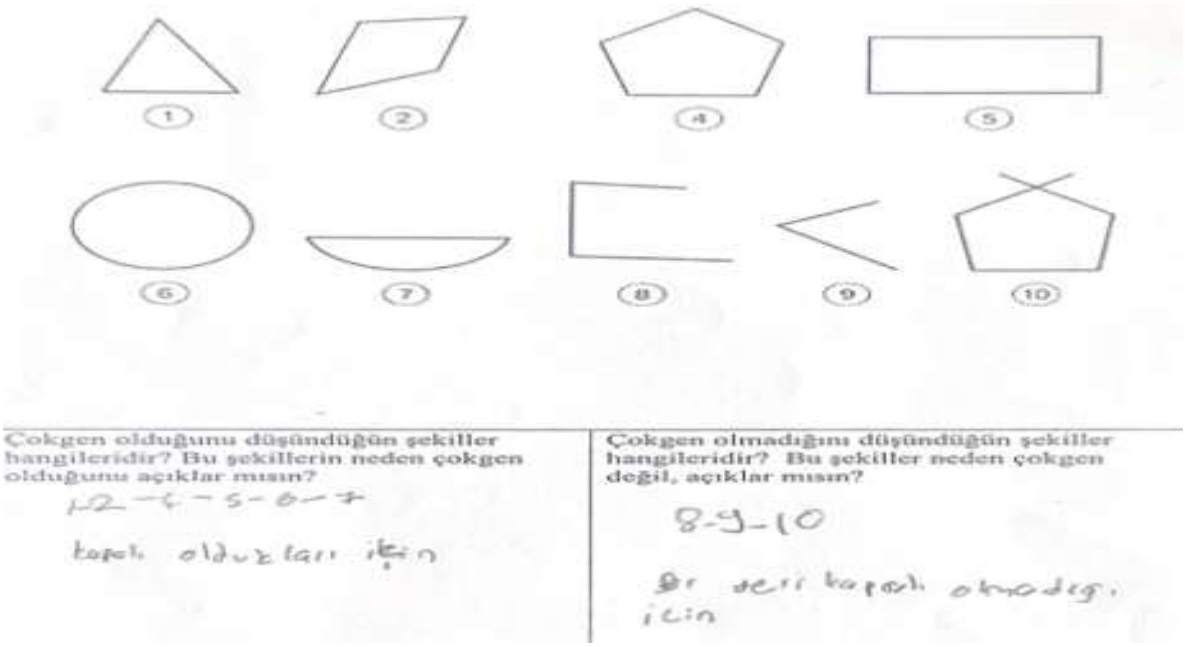


Figure 1

Example of student response

During the teacher intervention stage of HLT, the teacher did not provide definitions directly but instead used guiding questions to deepen students' justifications (e.g., What properties are required for a shape to be a polygon?). For example, it was observed that a student who had

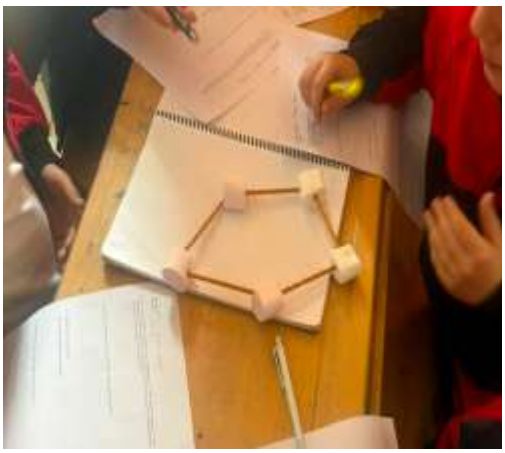
difficulty understanding the concept of a polygon believed that a polygon only needed to be closed (Figure 1). Thus, the inquiry process was preserved while learning was structured developmentally.

2. Conceptualization Activity

The conceptualization activity, which aimed to generate hypotheses in IBL, was implemented as the second activity. The HLT learning goal of the conceptualization activity was for students to construct different polygons using concrete materials and arrive at a definition of a polygon (Figure 2). Thus, it was expected that students would transition from an intuitive state to the stage of generating definitions. Through this group activity, students were expected to conceptualize the fundamental properties of polygons and the necessary and sufficient conditions for a shape to be a polygon.

In achieving the goal within HLT, concrete materials (e.g., geometry strips or sticks) were distributed to students. The question "What properties are required for a shape to be a polygon?" was opened for discussion based on the shapes they constructed. In this process, the students were guided to determine the necessary and sufficient conditions of the polygon concept and to develop correct and incorrect hypotheses. Within the scope of HLT, reasoning paths such as "sides must be equal/must be closed/sides must be linear" were anticipated.

In the teacher intervention stage of HLT, it was observed that some students were undecided about whether a circle is a polygon. It was anticipated that they would make errors that were not consistent with mathematical terminology when defining polygons (Figure 3). Therefore, incorrect hypotheses were written on the board, and necessary revisions were made. This ensured the continuation of the inquiry process.



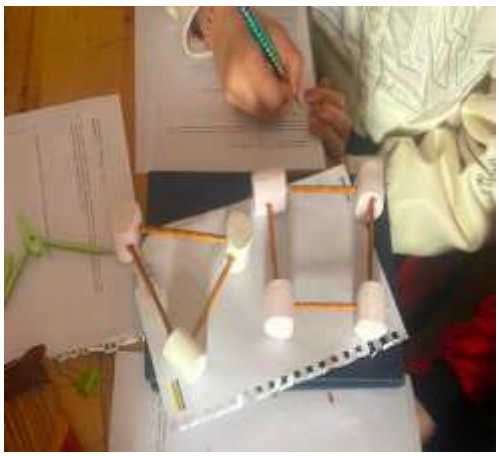


Figure 2

Examples from the implementation process

At the end of this stage, it was observed that the students' conceptual awareness regarding the concept of polygons improved significantly. They were able to clearly express distinguishing properties of polygons such as being closed, having linear sides, and having at least three sides.

However, for ten students, the formulation of the polygon definition in accordance with mathematical terminology was not yet fully achieved. These students used expressions based on everyday language such as “should not be round,” “should not be incomplete,” and “its sides should be connected” while defining polygons. This finding indicates that conceptual clarity was achieved for some students, but they had not yet reached a sufficient level in the use of mathematical language.

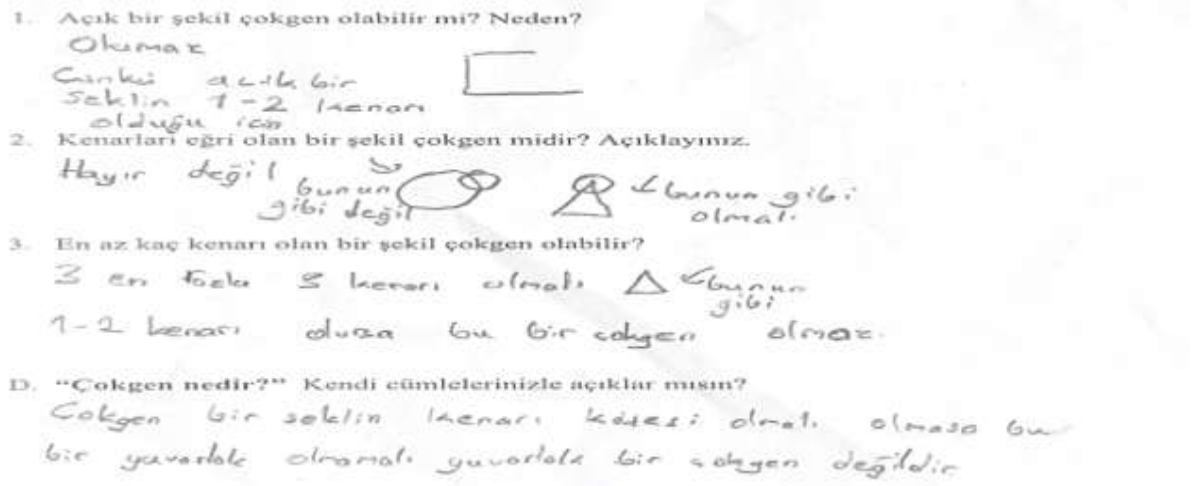


Figure 3

Example of student response sheet

3. Investigation Activity

The investigation activity of IBL, which includes exploration, experimentation, and data interpretation processes, was implemented as the third activity. The HLT learning goal of the investigation activity was for students to directly measure the side lengths and angle measures of different polygons using a ruler and a protractor, compare the data obtained, and, based on these comparisons, recognize the differences between regular and irregular polygons and define these concepts. At the same time, this activity aimed to help students realize that when deciding on the properties of a shape, they should use measurable mathematical properties rather than relying solely on visual intuition. In the activity, students were expected to classify polygons not only according to their visual features but also based on mathematical criteria (equality of sides and angles). In other words, the activity aimed to help them develop mathematical reasoning and generalization skills through measurement-based inquiry.

To achieve the HLT goal, the students were asked to calculate the side lengths and angle measures of given polygons using a ruler and a protractor. In addition, they were able to observe changes in side lengths and angle measures of regular and irregular polygons using the GeoGebra software.

In the teacher intervention stage of HLT, it was observed that the majority of students had difficulty using the ruler and protractor. In particular, technical errors were made in the use of the protractor and measurement accuracy. Therefore, errors arising from measurement were encountered in the activity sheets. At this point, teacher intervention was not carried out by

directly providing results but rather by offering technical guidance related to the measurement process. Thus, the students had some deficiencies in the processes of data generation and interpretation, and technical obstacles arose from the use of materials. Operational difficulties that were not anticipated in HLT but emerged during implementation showed that instructional flexibility was required**.**

4. Conclusion Activity

The conclusion activity of IBL, which requires drawing conclusions based on data, was implemented as the fourth activity. The HLT learning goal of the conclusion activity was to help students generalize the knowledge they had learned. In the activity, students were presented with open-ended prompts that questioned the conditions of the definition of polygons, targeted perceptual misconceptions, and required them to make measurement-based generalizations. Thus, the activity aimed to help students construct the concept of polygons in a recognition-based and justified manner.

To achieve the HLT goal, six open-ended questions requiring students to question what they had learned about polygons were posed. The students were asked to respond to these questions individually after group discussion. In the process, the students were expected to make generalizations and develop justified explanations using the conceptual knowledge they had constructed in previous stages.

In the teacher intervention stage of HLT, it was observed that some students continued to hold certain perceptual misconceptions. For example, some responded to the statement “Shapes that do not look regular are not polygons” with “Yes, I agree. Because a shape that does not look regular cannot be a polygon.” This situation indicates that, for some students, visually based thinking persisted. In addition, most students began to develop reasoning based on mathematical criteria. For example, to the question, “Is it sufficient for a polygon to have only equal sides for it to be a regular polygon?” they responded, “It is not sufficient. Because even if the sides are equal in length, the angle measures may be different. One side may not be a line segment.” This expression indicates that students began to distinguish between necessary and sufficient conditions and to develop justification based on mathematical criteria.

5. Discussion and Reflection Activity

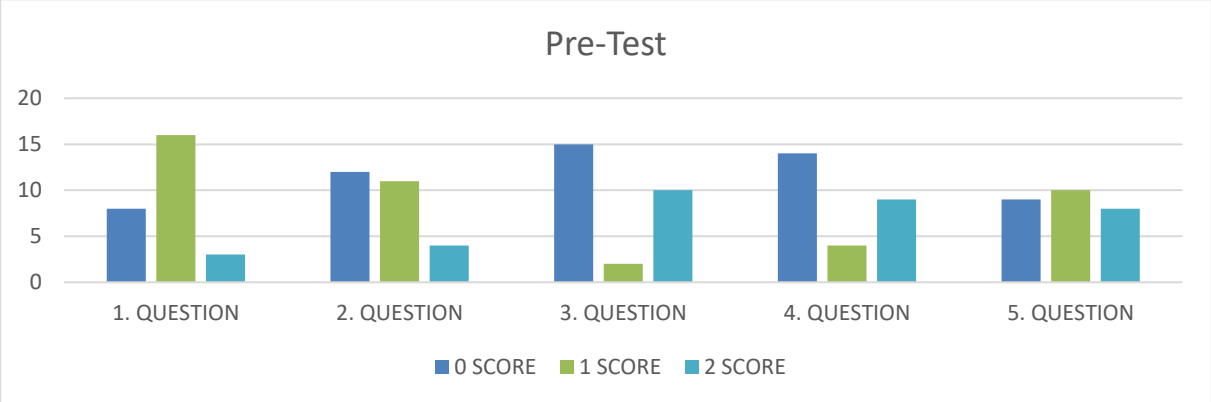
The discussion activity of IBL, which aims to reveal students’ communication and reflection skills, was implemented as the final activity. The HLT learning goal of the discussion activity was for students to express in writing the knowledge they had acquired throughout the process and express it in a holistic manner. Students were expected to integrate and express their thoughts regarding the definition of polygons, the differences between regular and irregular polygons, and the inferences they developed based on measurement. Thus, the activity aimed to enable students to demonstrate not only their conceptual knowledge but also how they construct and express it.

To achieve the HLT goal, the students were presented with the prompt, "Write what you have learned about polygons." The students' written responses to this question were analyzed to evaluate the level of conceptual learning and the use of mathematical language.

In the teacher intervention stage of HLT, the findings obtained showed that the majority of the students were able to express the knowledge they had learned in a clear and understandable manner in their written statements. They included the fundamental properties of polygons (e.g., being closed, having linear sides, and having at least three sides) and the distinguishing properties of regular polygons (e.g., equality of sides and angles) in their written expressions. This indicates that the students were able to integrate and express the conceptual knowledge they had constructed in previous stages.

Evaluatıon of the Actıvıty

To evaluate the activities prepared based on the integration of IBL and HLT, a comparison was made using the data collection instrument administered as a pre-test and post-test. The results of the pre-test and post-test are presented in Graph 1 and Graph 2, respectively.



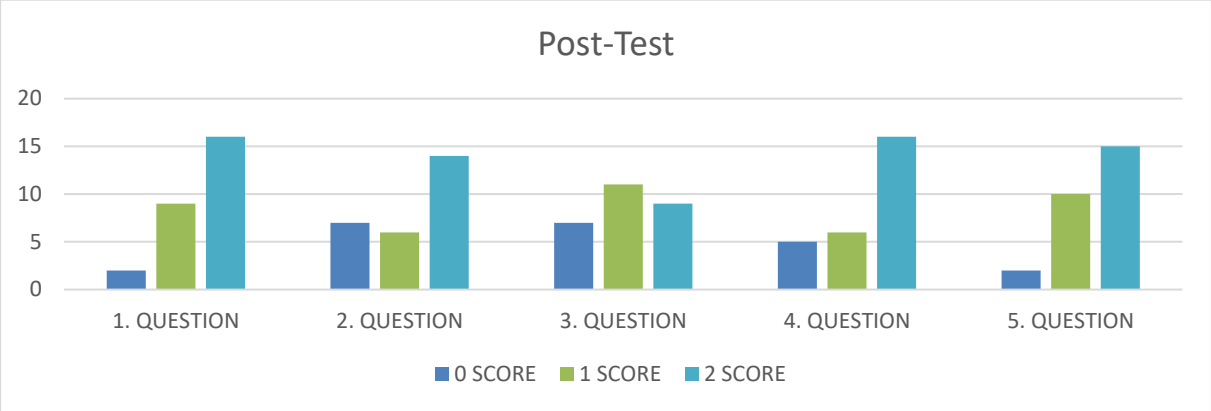
Graph 1

Pre-test results

After the IBL and HLT integration activities, the participating students' conceptual understanding of polygons improved. The responses given by the students to Question 1 revealed that the majority understood the concepts related to the basic properties of polygons and the relationships among these concepts. Students' responses to Question 5 indicate that misconceptions about whether a circle is a polygon were largely corrected; however, some errors in mathematical expressions persisted. Questions 2 and 3 assessed students' ability to apply the concepts they had learned about polygons. The students' responses to these questions showed improved application skills regarding polygons, but their transformation skills still needed improvement. Lastly, based on the students' responses to Question 4, some students had skills that needed to be improved in the use of a protractor and a ruler.

The Wilcoxon signed-rank test was used to analyze whether there was a statistically significant difference between the students' pre-test and post-test scores. First, the total scores obtained by

each student on the pre-test and post-test were calculated. Table 1 presents information regarding the results of this analysis.



Graph 2

Post-test results

Table 1.

Wilcoxon Signed-Rank Test Results for Students' Pre-Test and Post-Test Total Scores.

Pretest–posttest	n	$Mean\ rank$	$Sum\ of\ the\ ranks$	z	p
Negative rank	0	.00	.00	−4.136	.001
Positive rank	22	11.50	253.00		
Tie	5	–	–		

A statistically significant difference was found between the students' pre-test and post-test scores $[z = -4.136, p < .05]$. Considering the mean ranks and the sums of the ranks, the observed difference was in favor of a positive rank, that isi.e., the post-test scores.

Dıscussıon and Conclusıon

This study aimed to enable fifth-grade students to meaningfully discover and learn concepts related to polygons and the relationships among these concepts through activities prepared in accordance with the integration of IBL and HLT. In the study, five activities structured in accordance with the integration of IBL and HLT were implemented. The data collection instrument prepared to evaluate the activities was administered to the students before and after the implementation. The results show that activities prepared based on the integration of IBL and HLT improved students' conceptual understanding of polygons. During the implementation, it is recommended that both the positive aspects of the process and areas requiring improvement be evaluated at certain points.

The findings from the initial orientation activity revealed deficiencies in students' understanding of the basic properties of polygons. Rizki and Amir (2025) emphasized that activities designed in accordance with the integration of IBL and HLT should remain flexible and adaptable to address differences in students' readiness levels and prior knowledge. The findings from the conceptualization activity indicated that students initially explained concepts using everyday language rather than formal mathematical terminology. In the teacher intervention stage of HLT, necessary interventions were made for students who were insufficient in using mathematical terminology. This finding from the conceptualization activity indicates that in the IBL process, conceptual meaning is constructed before formal definition. When evaluated within the HLT framework, this finding can be interpreted as indicating that students recognized the necessary properties but had not yet transitioned to the stage of formulating these properties in accordance with mathematical terminology. This finding is consistent with the developmental progression anticipated in HLT. The realization of conceptual clarity before mathematical formulation is consistent with the nature of IBL. Therefore, mathematical precision in formulating the definition was addressed in a later stage.

In the investigation activity, students were expected to recognize the concepts of regular and irregular polygons. This activity showed that the development of conceptual understanding depended not only on cognitive processes but also on proficiency in using measurement tools. This finding emerged as an aspect that should be considered in the HLT implementation process. Another finding obtained from the investigation activity indicates that some students tended to classify regular and irregular polygons based on visual perception. In addition to distinguishing polygons visually, relationships should be established based on the concept definition (Güven et al., 2019). For the proper progression of the process, future studies should examine whether students use the materials correctly during the teacher intervention stage of HLT. Findings from the discussion and conclusion activities showed that the majority of students were able to reach generalizations regarding polygons. While students partially maintained perception-based misconceptions, they also began to develop a more advanced conceptual structure based on measurement and reasoning. This situation reveals that conceptual learning is not linear but a gradual and multi-layered process. Consistently, Rizki and Amir (2025) reported that in the integration of IBL and HLT, low-achieving students benefitted more from concrete materials and intensive guidance, while high-achieving students developed more effectively in questioning, exploring spatial relationships, and generalizing concepts.

In conclusion, activities prepared in accordance with the integration of IBL and HLT had a positive effect on students' discovery and learning of concepts related to polygons and the relationships among these concepts and also contributed to the development of their conceptual understanding of polygons. However, for some students, the persistence of perceptual and visual intuitions should be considered during teacher intervention in the HLT implementation process. When teachers understand learning trajectories, it is important for them to understand both mathematics and students' thinking and learning processes, as well as how to support them effectively (Clements & Sarama, 2009).

References

Akker, J. J. H., Gravemeijer, K., McKenney, S., & Nieveen, N. (Eds.). (2006). *Educational design research*. Routledge, Taylor & Francis Group. https://doi.org/10.4324/9780203088364

Argün, Z., Arıkan, A., Bulut, S., & Halıcıoğlu, S. (2020). *Temel matematik kavramlarının künyesi*. Ankara: Palme Yayınevi.

Bernabeu, M., Moreno, M., & Llinares, S. (2021). Primary school students' understanding of polygons and the relationships between polygons. *Educ Stud Math 106*, 251–270. https://doi.org/10.1007/s10649-020-10012-1

Baroody, A. J., Clements, D. H., & Sarama, J. (2022). Lessons learned from 10 experiments that tested the efficacy and assumptions of hypothetical learning trajectories. *Education Sciences, 12*(3), 195. https://doi.org/10.3390/educsci12030195

Clements, D. H., & Battista, M. T. (1992). Geometry and spatial reasoning. *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics,* 420-464.

Clements, D. H., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach.* Routledge.

Confrey, J. & Maloney, A. (2010). The construction, refinement, and early validation of the equipartitioning learning trajectory. *ICLS, 1*, 968-975.

Eroğlu, D., Camci, F., & Tanişli, D. (2019). Hypothetical learning trajectory to the development of sixth grade students' knowledge about fractions and addition-subtraction in fractions. *Pamukkale University Journal of Education, 45*, 116-143.

Eroğlu, D. & Tanışlı, D. (2020). Tahmini öğrenme yollarının uygulanması sürecinde matematiköğretmenlerinin çoklu temsil kullanımlarının gelişimi. *Cumhuriyet International Journal of Education, 10*(1), 299-329.

Güven, B., Öztürk, T., & Bülbül, B. Ö. (2019). *Geometri öğretimi*. İçinde G. Hacıömeroğu & Kamuran T. (Eds.) Matematik Öğretiminin Temelleri Ortaokul (ss.169-214). Ankara: Anı.

Jia, B., & Zhu, Z. (2025). Mathematical knowledge learning trajectories: an ınternational comparative study. *SAGE Open*, *15*(3), 21582440251375799.

Lazonder, A. W. (2013). *Inquiry learning.* In Handbook of research on educational communications and technology (pp. 453-464). New York, NY: Springer New York.

Ministry of National Education [MoNE] (2024a). Ortaokul matematik dersi öğretim programı (5, 6, 7 ve 8. Sınıflar): Türkiye Yüzyılı Maarif Modeli.

MoNE (2024b). Türkiye Yüzyılı Maarif Modeli Öğretim Programları Ortak Metni.

MoNE (2024c). *Ortaokul ve imam hatip ortaokulu 5. sınıf matematik ders kitabı 1. kitap*. Devlet Kitapları Müdürlüğü.

Öztürk, M. (2020). *Çokgenler ve Öğretimi.* İçinde E. Ertekin & M. Ünlü (Eds.) Geometri ve Ölçme Öğretimi (ss.252-269). Ankara: Pegem.

Pedaste, M., Mäeots, M., Siiman, L. A., De Jong, T., Van Riesen, S. A., Kamp, E. T., ... & Tsourlidaki, E. (2015). Phases of inquiry-based learning: Definitions and the inquiry cycle. *Educational Research Review, 14*, 47-61.

Rizki, F. A., & Amir, M. F. (2025). Hypothetical learning trajectory based on inquiry learning to find the volume of space. *Matematika dan Pembelajaran, 13*(1), 19-42.

Sztajn, P., Confrey, J., Wilson, P. H., & Edgington, C. (2012). Learning trajectory-based instruction: Toward a theory of teaching. *Educational Researcher, 41*(5), 147-156.

Simon, M. A. (1995). Reconstructing mathematics pedagogy from a constructivist perspective. *Journal for Research in Mathematics Education, 26*(2), 114-145.

Ulusoy, F. (2022). *Çokgenlerin öğretimi.* İçinde Z. Toluk-Uçar, R. Akkuş, Boz-Yaman, A. Duatepe-Paksu & S. Bulut (Eds.) Geometri Öğretim Bilgisi (ss.133-180). Ankara: Pegem.

Wiryanto, Habibie, R. K., & Nurlaily, V. A. (2024). Hypothetical learning trajectory strategy in ethno-realistic mathematics education: an exploration of Damar Kurung. *Jurnal Elementaria Edukasia, 7*(3), 3062-3071.

Vasuki, M., Celestin, M., & Kumar, A. D. (2016). Exploring the role of inquiry-based learning in mathematics education. *Mathematics Education, 2*(2), 193-206.

Zehir, K. & Zehir, H. (2022). *Çokgenlerin öğretimi*. İçinde T. Ağırman-Aydın & B. Küçük-Demir (Eds.) Geometri ve Öğretimi (ss. 79-101). Ankara: Pegem.

Zhu, Z. (2023). International comparative study of learning trajectories based on TIMSS 2019 G4 data on cognitive diagnostic models. *Front. Psychol. 14*, 1241656. https://doi.org/10.3389/fpsyg.2023.1241656

APPENDIX-1. Instructional design for the integration of inquiry-based learning and hypothetical **learning trajectory**

HLT Phases	**Learning Outcome**	**Achieving the Objective**				**Teacher Intervention**
		Activity	**Materials**	**Expected Student Responses**	**Possible Misconceptions**	
IBL Phases						
Orientation	Recognizing polygons	Various visuals consisting of a triangle, square, pentagon, circle, open broken line, and self-intersecting shapes are distributed to students. Students are asked to examine these shapes and answer the question "Which ones are polygons? Why?" with justification. During this process, a classroom discussion environment is created to encourage students to share and defend their ideas.	Visual shapes Geometry strips Pictures	Students are expected to separate shapes according to their visual properties; distinguish shapes whose sides are line segments and are closed; and express why shapes that are polygons are polygons and why those that	Mistaking a circle for a polygon Accepting shapes with open sides as polygons	Guidance through the question "What properties are required for a shape to be a polygon?" Helping students recognize the properties of the polygon concept through incorrect examples (e.g., circle, non-closed shape, self-intersecting shape)

				are not are not polygons.		
Conceptualization (Questioning-Hypothesis Generation)	Forming hypotheses about the properties of polygons	Students are asked to construct a definition of polygons through group work and to build different polygons using various concrete materials. Students determine the basic properties of the polygons they construct through discussion. It is aimed that they reach the necessary and sufficient conditions of the polygon concept.	Geometry strips Sticks	Students are expected to define polygons and develop hypotheses about polygons; e.g., sides must be equal, must be closed, and sides must be linear. They produce correct/incorrect hypotheses.	Not accepting shapes that do not look regular as polygons Thinking that a triangle is not a polygon Being unable to decide whether a circle is a polygon	Writing incorrectly generated hypotheses on the board and testing them again through class discussion
Investigation (Exploration-Experimentation-	Discovering regular polygons	Students are expected to directly measure the side lengths and angle measures of different polygons using a ruler and a protractor, compare the data obtained, and form generalizations about the concepts of regular and irregular	Protractor Ruler	Students may tend to classify polygons only according to visual features, independently of the objective of classifying	Assuming that if a shape looks regular, it is definitely a regular polygon Assuming that having equal side lengths alone is	Encouraging students to make inferences that support measurement-based inquiry, mathematical reasoning, and generalization skills Helping students recognize the properties

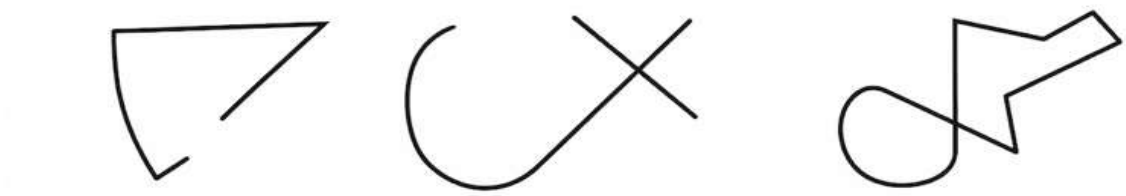
Data Interpretation)		polygons as a result of the comparison.		polygons based on mathematical criteria (equality of sides and angles).	sufficient for a regular polygon Thinking that only a square is a regular polygon Errors in using a ruler and a protractor	of regular polygons using GeoGebra "Can a square also be a rectangle? Is a rectangle a regular polygon? Why?"
Conclusion (Generalization-Comparison)	Generalizing concepts related to polygons	Students are presented with an open-ended prompt that questions the conditions of the polygon definition, target perceptual misconceptions, and require measurement-based generalization. It is aimed that students construct the concept of polygons in a recognition-based and justified manner.		Students' generalizations are written on the board.	Limiting generalizations only to specific cases	Discussing incorrect generalizations as a class and emphasizing correct ones
Discussion and Reflection	Evaluating the learning process and correcting misconceptions	Students are expected to express in writing the knowledge they have acquired about polygons. This activity provides the opportunity to evaluate students' understanding of the polygon		Students' recognize misconceptions and begin to defend the correct concept	Persistence of resistant misconceptions based on visual perception	Providing comparative examples and asking reflection questions to help students recognize their own mistakes

		definition, the distinction between regular and irregular polygons, and their measurement-based inferences in a holistic manner.				

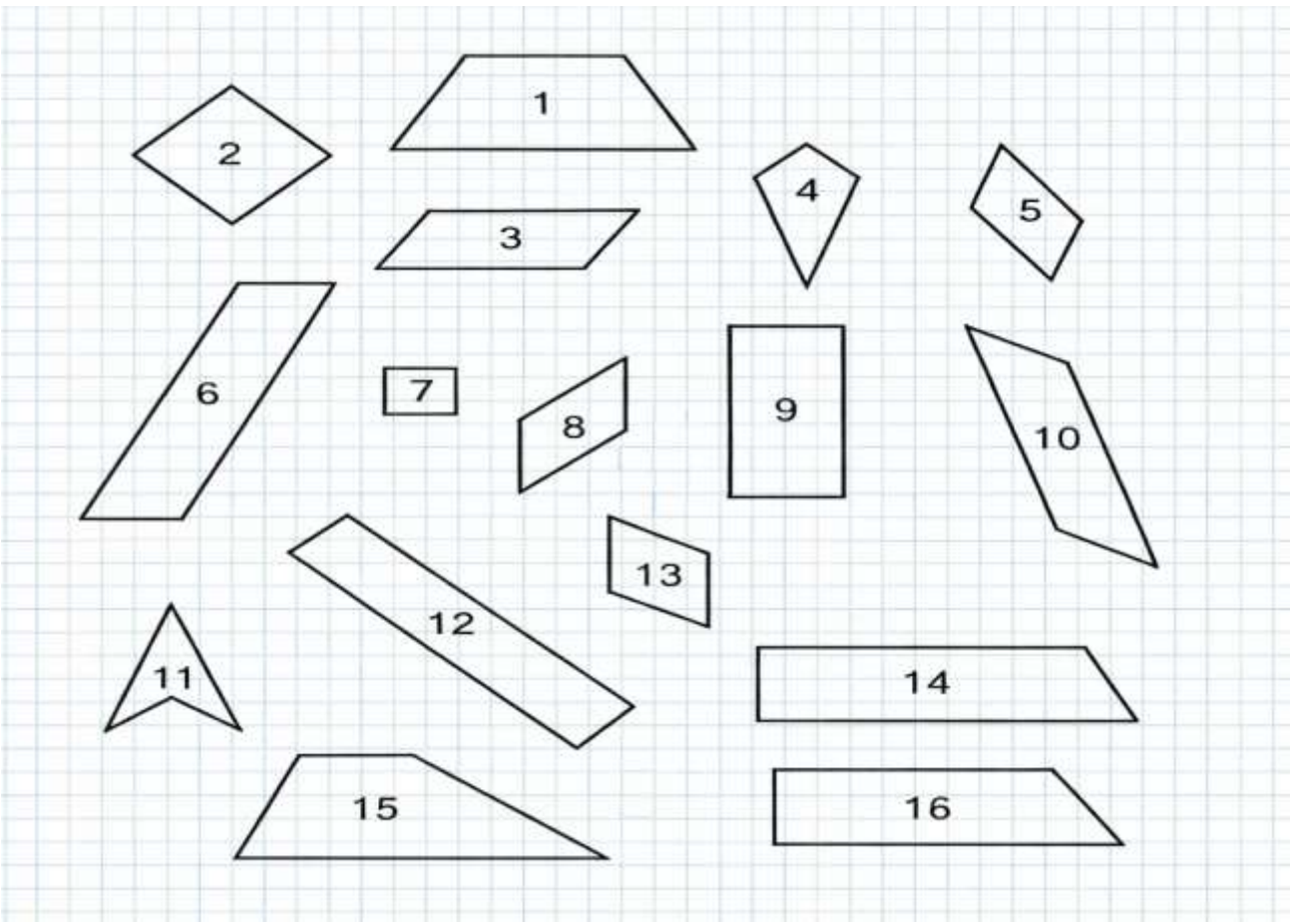
APPENDIX 2. Data Collection Instrument

1. What properties must a shape have in order to be classified as a polygon? Draw a polygon and explain why the shape you have drawn qualifies as a polygon.

2. The shapes below are not polygons. Can you explain what needs to be done to transform these shapes into polygons? Then, draw the polygonal versions of these shapes.



3. Using your protractor and ruler, can you identify which of the following shapes are regular polygons? Explain in your own words why the shapes you selected are considered regular polygons.

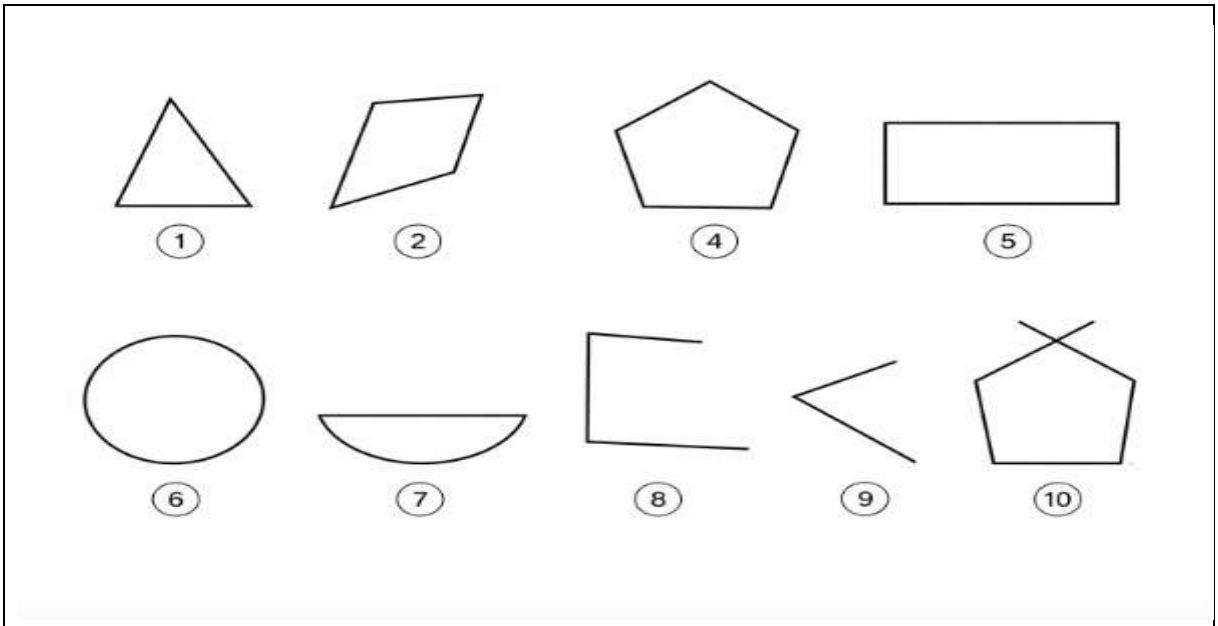


4. Place seven points on the paper below. Connect some of these points in pairs using line segments. Then, carefully examine the resulting figure. Do you think this figure is a polygon? Explain your reasoning in your own words. How would you name the shape you have created?

5. A student states, “A circle is a polygon.” Can you explain whether this statement is correct or incorrect?

APPENDIX-3. Orientation Activity

“Which of the following shapes are polygons?”



Which shapes do you think are polygons? Can you explain why these shapes are polygons?	Which shapes do you think are not polygons? Can you explain why these shapes are not polygons?

Conceptualization Activity

A. Using the sticks given to you, create different shapes with your group members. First, create a few shapes freely. Examine the shapes you have created. Which of the shapes you created do you think are polygons?

B. What conditions must a shape satisfy in order to be a polygon?

1. ……

2. ……

3. ……

4. ……

5. ……

C. Answer the following questions:

1. Can an open shape be a polygon? Why?

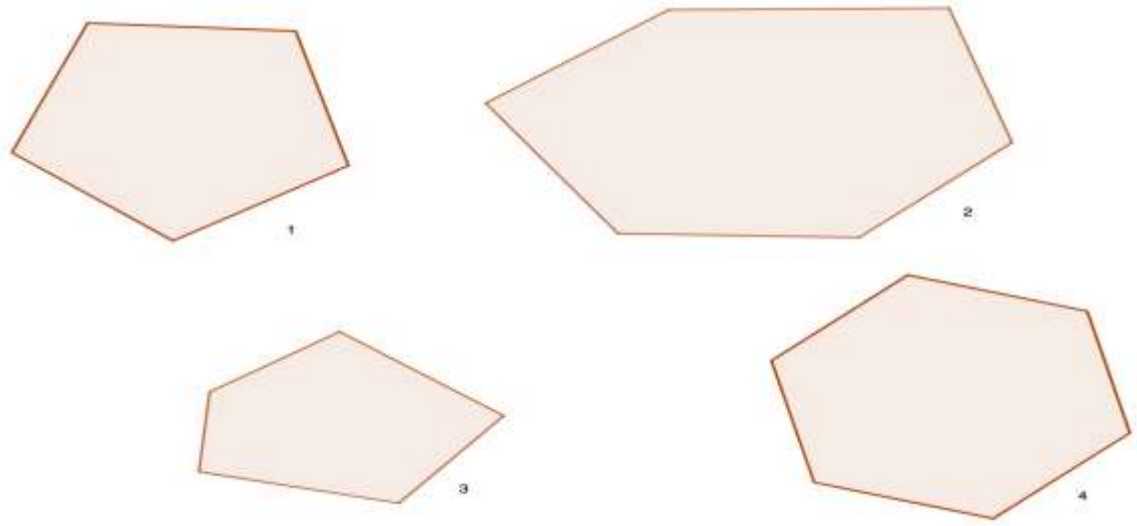
2. Is a shape with curved sides a polygon? Explain.

3. What is the minimum number of sides a shape must have to be a polygon?

D. "What is a polygon?" Can you explain in your own words?

Investigation Activity

Using the ruler and protractor provided to you, measure the side lengths and interior angle measures of the following polygons and record the measurement results in the table below.



Polygon	Side Lengths (cm)	Angle Measures	Are the Sides Equal?	Are the Angles Equal?
1. Polygon				
2. Polygon				
3. Polygon				
4. Polygon				

1. Polygons whose sides and angles are equal are called **……...**

2. Polygons whose sides and angles do not have to be equal are called **……...**

Discussion Activity

Discuss the following questions with your group members and answer them. Write your answers with justification.

1. Can a shape that has only one of the following properties be a polygon? Explain your reason.

- Being closed
- Having at least three sides
- Having sides that are line segments

2. A student says: "Shapes that do not look regular are not polygons." Do you agree with this student? Explain your reason.

3. What must we do to transform an open shape into a polygon? Please explain.

4. Is it sufficient for a polygon to have only equal sides for it to be a regular polygon? Explain your reason.

5. Give an example of a polygon that has equal sides but is not regular. Explain why this polygon is not regular.

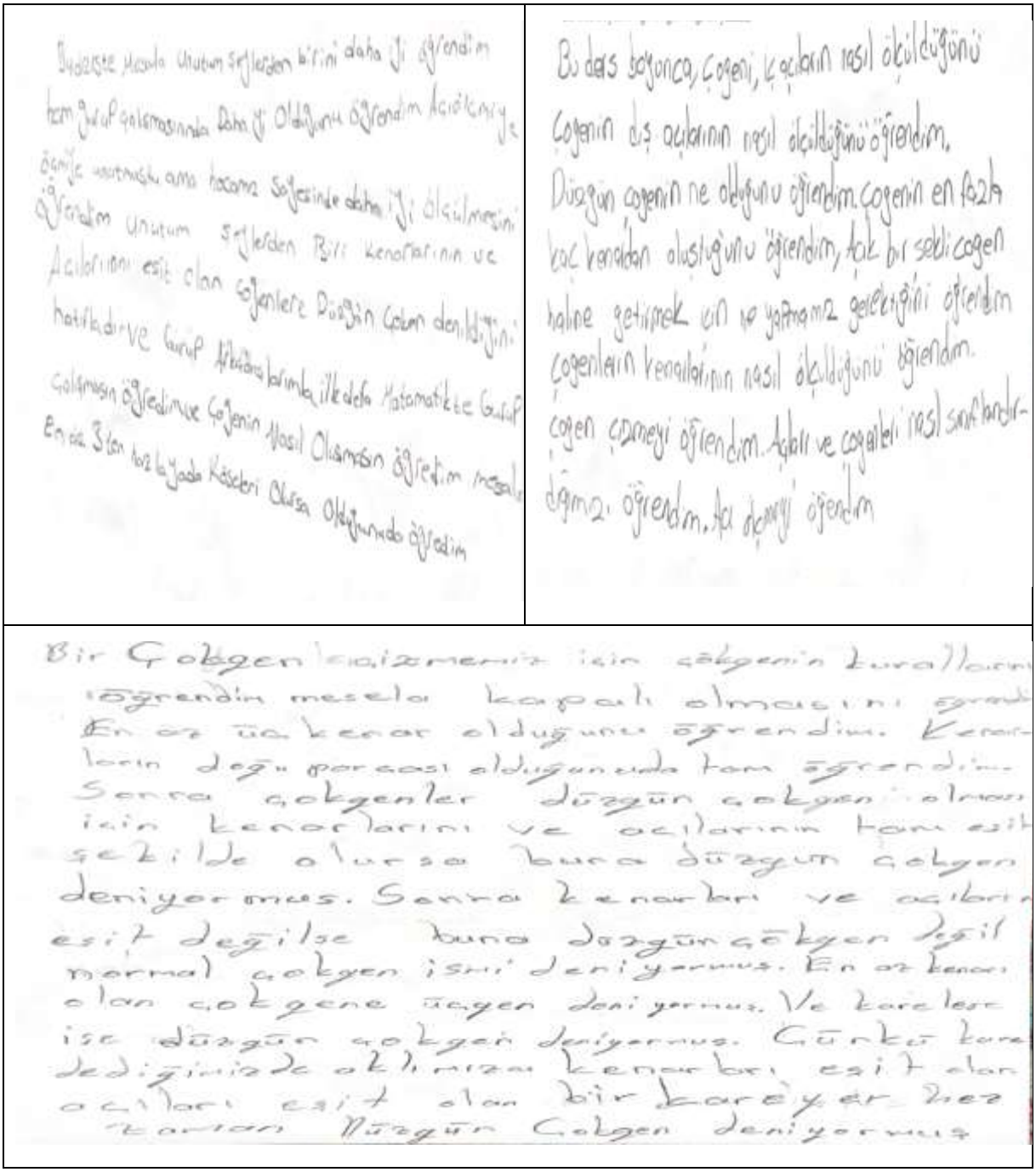
6. How did the measurements you made using a ruler and protractor help you classify polygons? Explain.

Conclusion Activity

Write what you have learned about polygons.

Figure 4.

Examples of student responses to the discussion and reflection activity


Bu ders boyunca çokgen, iç açıların nasıl ölçüldüğünü
çokgenin dış açılarının nasıl ölçüldüğünü öğrendim.
Düzgün çokgenin ne olduğunu öğrendim çokgenin en az
kaç kenardan oluştuğunu öğrendim, açık bir şekli çokgen
haline getirmek için ne yapmam gerektiğini öğrendim
çokgenlerin kenarlarının nasıl ölçüldüğünü öğrendim.
çokgen çeşitlerini öğrendim. Açılı ve çokgenleri nasıl sınıflandır-
dığımı öğrendim. Açı çeşitlerini öğrendim
Bir çokgen oluşabilmesi için çokgenin kurallarını
öğrendim mesela kapalı olmasını gerek
En az üç kenar olduğunu öğrendim. Kenar-
ların doğru parçası olduğunda tam öğrendim.
Sonra çokgenler düzgün çokgen olması
için kenarlarını ve açılarının tam eşit
şekilde olursa buna düzgün çokgen
deniyormuş. Sonra kenarları ve açıları
eşit değilse buna düzgün çokgen değil
normal çokgen ismi deniyormuş. En az kenarı
olan çokgene üçgen deniyormuş. Ve kareler
ise düzgün çokgen deniyormuş. Çünkü kare
dediğimizde aklımıza kenarları eşit olan
açıları eşit olan bir kare gelir. Hem
kenarları düzgün çokgen deniyormuş